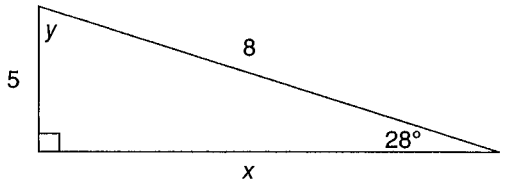
5.5

Trigonometric Ratios

Decide whether each trigonometric ratio could be used to find the missing side or angle.



1. To find *x*:	Circle one:		Explain your choices:
a. $\sin 28 = \frac{x}{8}$	Yes	No	
b. $\cos 28 = \frac{x}{8}$	Yes	No	
c. $\tan 28 = \frac{x}{5}$	Yes	No	
d. $\tan 28 = \frac{5}{x}$	Yes	No	

2. To find *y*:	Circle one:		Explain your choices:
a. $y = \frac{\frac{5}{8}}{\sin}$	Yes	No	
b. $y = \sin^{-1}\left(\frac{5}{8}\right)$	Yes	No	
c. $y = \frac{\frac{5}{8}}{\cos}$	Yes	No	
d. $y = \cos^{-1}\left(\frac{5}{8}\right)$	Yes	No	

Teacher Notes: Trigonometric Ratios

Questions to Consider About the Key Mathematical Concepts

When solving for unknown side lengths or angle measures in right triangles, can students recognize correct trigonometric ratios to use? To what extent do they

- make sense of which ratio(s) to use based on the hypotenuse and the adjacent and opposite sides of a right triangle?
- reason about whether to use sine, cosine, or tangent to find an unknown measure?
- describe how trigonometric ratios can be used to create a proportion to find an unknown side length or angle measure?

Common Core Connection (CCSS.Math.Content.HSG-SRT.C.8)

Grade: High School

Domain: Geometry

Cluster:

Define trigonometric ratios and solve problems involving right triangles.

C8. Use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems.

Uncovering Student Understanding About the Key Concepts

Using the Trigonometric Ratios Probe can provide the following information about how the students are thinking about ratios with right triangles.

Do they

- know the correct ratios for sine, cosine, and tangent?

OR

- determine which legs of the triangle are the opposite and the adjacent sides based on a particular angle?

Do they

- confuse which side lengths to use in the ratios?

OR

- confuse the opposite and adjacent sides, or not realize they change based on which angle is being used as a reference angle?

Do they

- use inverse trigonometric functions to find angle measures ($\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$)?

OR

Do they

- not realize that inverse functions are needed to find missing angle measures, oftentimes "getting rid of" the word *sin*, *cos*, or *tan* by dividing both sides of the equation by it (using it as a variable)?

Exploring Excerpts From Educational Resources and Related Research

Areas of consideration:

Trigonometry is an area of mathematics that students believe to be particularly difficult and abstract compared with the other subjects of mathematics. Three generalizations were made because of their relationship to Piaget's description of formal operations that could be drawn from the study on misconceptions. These three generalizations are

1. Many misconceptions are related to a concept that produces a mathematical object and symbol. For example, sine is a concept and symbol of trigonometric functions.
2. Many misconceptions are related to process: the ability to use operations, for example, as representing the result of calculation of sin 300° and value of sin 300°.
3. Many misconceptions are related to procept that is, the ability to think of mathematical operations and object. Procept covers both concept and process. For example: sin x is both a function and a value. In addition to this, Gray and Tall (1994) asserted that "procedural thinking," that is, the ability to think of mathematical operations and object as procept, is critical to the successful learning of mathematics. (Gur, 2009, p. 68)

Surveying the Prompts and Selected Responses in the Probe

The Probe consists of two selected response items, each related to a common figure. The prompts and selected responses are designed to elicit understandings and common difficulties as described in the following table.

If a student chooses	*It is likely that the student*
1. a. No, b. Yes, c. No, and d. Yes 2. a. No, b. No, c. No, and d. Yes (correct response)	• knows and understands the trigonometric ratios and can set up equations to find unknown side lengths (Question 1) and angle measures (Question 2). In the case of Question 2, the student realizes that the inverse function needs to be used to find the unknown angle measure. [See Sample Student Responses 1 and 2] *Look for indication of the student's understanding in the written explanations of how the student got the answer.*
All other responses	• has not learned the trigonometric ratios or confuses which side lengths to use based on what is given. Students often mix up the adjacent and opposite sides. In Question 2, students often do not understand how to find an angle measure or incorrectly apply the inverse function. Many students see the trigonometric functions (sin, cos, and tan) as variables and use division to "undo" them. [See Sample Student Responses 3 and 4]

Teaching Implications and Considerations

Ideas for eliciting more information from students about their understanding and difficulties:

- What do the words *hypotenuse, adjacent,* and *opposite* mean?
- Based on the angle that is 28°, which side is opposite it? What is the length of that side?
- Based on the angle that is 28°, which angle is adjacent to it? What is the length of that side?
- Based on the angle that is y, which side is opposite it? What is the length of that side?
- Based on the angle that is y, which angle is adjacent to it? What is the length of that side?
- How is finding an angle different from finding a side?
- What are the ratios for sine, cosine, and tangent? How are they similar? How are they different?
- What does sin y (or cos y or tan y) mean? Does it mean multiplication because they are right next to each other? (No)
- How do we "get rid of" the "sin" in the equation $\sin x = \left(\frac{4}{5}\right)$?
- What does an inverse trigonometric function do? When do you use it?

Ideas for planning instruction in response to what you learned from the results of administering the Probe:

- Allow students many opportunities to explore right triangles thoroughly, with paper and pencil and with technology too.

- When drawing triangles, orient the triangles differently so students get a better understanding of opposite and adjacent sides. Use triangles that are labeled other than ΔABC.
- Have students ask themselves what each side is called (hypotenuse, adjacent, or opposite) in reference to a specific angle.
- SOH CAH TOA is a great way for students to remember the trigonometric ratios, especially if it is told via an exciting story of the land of the Trigs and chief SOH CAH TOA!
- As inverse trigonometric functions are probably new concepts for most students, provide various opportunities to explore "undoing" sin, cos, and tan with inverse functions. This will become second nature to them if they have many opportunities to use it and practice with it.

Sample Student Responses to Trigonometric Ratios

Responses That Suggest Understanding

Sample Student Response 1

Probe Item 1: Student chose No for a and c and Yes for b and d. I used SOH CAH TOA to decide which ones are correct. Based on the angle we are given (28°), the opposite side is "5," the adjacent side is "x," and of course the hypotenuse is "8." Using these and the rules (SOH CAH TOA), the only two that can be used is b and d. $\sin 28 = \frac{5}{8}$, which does not include x; $\tan 28 = \frac{5}{x}$, not $\frac{x}{5}$; and $\cos 28 = \frac{x}{8}$.

Sample Student Response 2

Probe Item 2: Student chose No for a, b, and c and Yes for d. As we are looking for y, we cannot have another unknown so cannot use the opposite side to angle y, which is x. This eliminates sin and tan. As we are looking for an unknown angle, we will need to use an inverse function, and choice d works! For choice c, we divide by cos instead of taking the inverse.

Responses That Suggest Difficulty

Sample Student Response 3

Probe Item 1: Student chose Yes for a, b, c, and d. You can solve for x in all of the equations.

Probe Item 2: Student chose Yes for a and c and No for b and d. To solve for y, you have to get it alone by dividing each side by what is next to it (sin or cos). Not sure what that -1 thing is.

Sample Student Response 4

Probe Item 1: Student chose No for c and d and Yes for a and b. c and d are not right because the adjacent side is 8, and there is no 8 in either equation.