Tools for Facilitating Meaningful Mathematics Discourse

This article explores facilitating meaningful mathematics discourse, one of the research-based practices described in *Principles to Actions: Ensuring Mathematical Success for All*. Two tools that can support teachers in strengthening their classroom discourse are discussed in this, another installment in the series.

Copyright © 2019 The National Council of Teachers of Mathematics, Inc. www.nctm.org.
All rights reserved. This material may not be copied or distributed electronically or in any other format without written permission from NCTM.

> Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
>
> *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014, p. 29)

Michael D. Steele

Teaching in ways that support meaningful mathematics discourse among students can be a significant challenge. Teachers and students alike can easily gravitate toward the traditional Initiate-Response-Evaluation (IRE) pattern, in which the teacher poses a question, a student responds, and the teacher evaluates the correctness of that response or otherwise provides feedback (Mehan 1979). The recommendations of *Principles to Actions*, backed by decades of research, show us that when students discuss and make mathematical meaning together, they learn more and better mathematics (e.g., Carpenter, Franke, and Levi 2003; Lampert and Cobb 2003; Michaels, O'Connor, and Resnick 2008; Smith and Stein 2018).

In the cycle of planning, teaching, and assessing, we as teachers might be tempted to focus on discourse primarily within the teaching phase. Shifting to a more discourse-centered classroom certainly means changing our interactions with students during the lesson. Those interactions, however, are more effective when we use tools to plan for discourse. These tools can also help us better assess student thinking and learning in a student-centered classroom. In this article, I draw on two research-based tools, *5 Practices for Orchestrating Productive Discussions* (Smith and Stein 2018) and the Teacher Discourse Moves explored in *Mathematics Discourse in Secondary Classrooms* (Herbel-Eisenmann et al. 2017). I then discuss how these two sources provide teachers with tools to strengthen discourse through planning, teaching, and assessing.

THE 5 PRACTICES: FOCUS ON PLANNING AND TEACHING

Smith and Stein (2018) identify 5 Practices for orchestrating productive mathematics discussions: *anticipating* likely student responses, *monitoring* student thinking while they work on the task, *selecting* particular students to present their mathematical work to the whole class, *sequencing* the student responses that will be displayed, and *connecting* different students' responses and connecting the responses to key mathematical ideas. These practices provide teachers with a structure to plan for a discourse-rich lesson that in turn can result in stronger student discourse and clear student learning outcomes related to the instructional goal. I briefly illustrate how the 5 Practices operate using the Pay It Forward task (Boston et al. 2017) shown in **figure 1**.

The 5 Practices is a powerful planning tool that supports asking questions and steering discussions in mathematically fruitful ways during teaching. The 5 Practices bring predictability and direction to classroom discourse that creates room for principled improvisation within a specified mathematical trajectory. **Table 1** shows a planning and monitoring tool (adapted from Smith and Stein 2018) that can supports engaging with the 5 Practices before and during a lesson. A goal for Pay It Forward might be for students to represent an exponential relationship using multiple representations and to generalize that relationship. With that goal in mind, let's examine the ways in which using the planning and monitoring tool can support stronger classroom discourse.

Fig. 1 The Pay It Forward task was explored by students (Boston et al. 2017, p. 10).

The Pay It Forward Task

In the movie *Pay It Forward*, a student, Trevor, comes up with an idea that he thinks could change the world. He decides to do a good deed for three people, and then each of the three people would do a good deed for three more people, and so on. He believes that before long, there would be good things happening to billions of people. At stage 1 of the process, Trevor completes three good deeds. How does the number of good deeds grow from stage to stage? How many good deeds would be completed at stage 5? Describe a function that would model the Pay It Forward process at any stage.

When we use a task with multiple solution paths that may be discussed and explained by students in diverse ways, it can be challenging to figure out how to respond to student thinking in the moment. The first three columns of **table 1**'s planning and monitoring tool are completed during planning. We first *anticipate* student thinking by noting the different approaches or lines of thinking (correct and incorrect) that students might take with the task. We will not think of every possible approach in advance, but documenting the most likely avenues of thinking will reduce our load as teachers in interpreting student work as we go. Alongside those solution strategies are tools for *monitoring* student thinking in the form of questions that assess and advance thinking toward the goal. Planning questions in advance provides you with tools to get students talking to one another, and to you, about their thinking as they are engaged in the task. For example, a group working on a table like that in row A might be asked, "Can you share with me the relationships you see in your table?" as a way to assess how students are making sense of it. "Can you add a column to your table that shows how you calculated each y-value?" would help advance student thinking, pressing them to consider the powers of 3 embedded in their calculations and move toward a generalization.

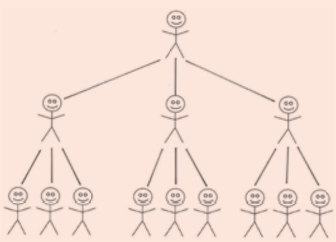
The remaining columns of **table 1** support the practices of *selecting* and *sequencing* student responses. During planning, teachers can identify the type and order of solutions that they would like to share in a whole-class discussion to build a mathematical storyline that moves toward the instructional goal. For example, a storyline for the Pay It Forward task could be to differentiate an exponential situation from a linear situation, to understand the nature of the underlying relationship as multiplicative, and that repeated multiplication can be represented as an exponential function. As we monitor student work, we can record which groups use which strategies, charting a course for the whole-class discussion of solutions. In our example, the teacher first shares an incorrect linear representation, allowing the class to discuss why $y = 3x$ may seem appropriate and why the generalization does not represent the number of deeds in each stage. Next, the tree diagram and table both show related representations of the correct number of deeds and can support discussion of the repeated pattern of multiplying the previous stage by 3 to get to the next one. This bridges to representing repeated multiplication as an exponential relationship and the sharing of $y = 3^x$, followed by discussion of the graph and how it shows the rate of change of the underlying function.

By thinking through the structure of the discussion in advance, teachers are free to focus on questions and comments that prompt *connecting* the solution strategies to one another and to the mathematical goal for the lesson. These purposeful questions should serve to make the mathematics visible and encourage reflection and justification (NCTM 2014), and they should relate to the mathematical goal. The work of connecting responses ties in the effective Mathematics Teaching Practices of *pose purposeful questions* and *establish mathematical goals to focus learning*, and moves the discussion beyond a simple "show and tell" of strategies.

In addition to supporting planning and teaching, the 5 Practices can support assessment. With the planning and monitoring tool in hand, teachers can note which students and groups use which strategies. Over time, those data can be used for formative and summative assessment. The tool helps chart a coherent course through a discourse-based lesson that focuses on moving students toward their mathematical goal. It also brings predictability to the implementation of tasks while allowing teachers to be thoughtful and responsive to emerging student thinking.

TEACHER DISCOURSE MOVES: KEEPING THE DISCOURSE MOVING

The 5 Practices chart the broader mathematical course of a lesson and help shape the storyline of a discourse-centered classroom. In addition, there are small-scale tools that teachers can use to promote productive and powerful discourse that both move the mathematics forward and attend to issues of student identity, agency, and power. Teacher Discourse Moves found in *Mathematics Discourse in Secondary Classrooms* (Herbel-Eisenmann et al. 2017) serve to support engagement and student-to-student discourse as well as position

<table>
<caption>Table 1 This planning and monitoring tool can be used for the Pay It Forward task.</caption>
<tr><th>Anticipated Solution Strategies</th><th>Assessing Questions</th><th>Advancing Questions</th><th>Who Has a Solution?</th><th>Order to Share</th></tr>
<tr><td>A. Table with stages and deeds
<table>
<tr><th>x stages</th><th>y (deeds)</th></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>2</td><td>9</td></tr>
<tr><td>3</td><td>27</td></tr>
<tr><td>4</td><td>81</td></tr>
<tr><td>5</td><td>243</td></tr>
</table></td><td>Can you share with me the relationships you see in your table?</td><td>Can you add a column to your table that shows how you calculated each y value?</td><td>Groups 3, 4</td><td>(Before 3 if time)</td></tr>
<tr><td>B. Table with stages and deeds, expanded multiplication
<table>
<tr><th>x stages</th><th></th><th>y (deeds)</th></tr>
<tr><td>1</td><td>3</td><td>3</td></tr>
<tr><td>2</td><td>3 × 3</td><td>9</td></tr>
<tr><td>3</td><td>3 × 3 × 3</td><td>27</td></tr>
<tr><td>4</td><td>3 × 3 × 3 × 3</td><td>81</td></tr>
<tr><td>5</td><td>3 × 3 × 3 × 3 × 3</td><td>243</td></tr>
</table></td><td>What does the times 3 column in your table represent?</td><td>How might we be able to represent the repeated multiplications of 3 in an equation?</td><td>Group 5</td><td>3</td></tr>
<tr><td>C. Tree diagram (at least 3 stages)
</td><td>How is your tree diagram changing at each stage?</td><td>It looks like continuing your diagram was not efficient. How can you represent growth past Stage 3 in a different way?</td><td>Groups 1, 2, 4</td><td>2</td></tr>
<tr><td>D. Symbolic equation $y = 3^x$</td><td>How does your equation represent what's happening at each stage?</td><td>How does the equation represent the rate at which the situation is changing at each stage?</td><td>Groups 3, 4, 5</td><td>4</td></tr>
<tr><td>E. Symbolic equation $y = 3x$</td><td>How did you write your equation?</td><td>What is the number of deeds in Stage 3? Does this match the result from your equation?</td><td>Group 2</td><td>1</td></tr>
<tr><td>F. Graph</td><td>How did you create your graph? How does it represent the values at stages 1, 2, and 3?</td><td>How quickly is the output value on the graph changing? How long do you think it would take to reach a million deeds?</td><td>Group 3</td><td>5 or as homework</td></tr>
</table>

students as authors of mathematical ideas. These moves, based on the work of Chapin, O'Connor, and Anderson (2013), appear in **table 2**.

The Teacher Discourse Moves can help on a small scale to keep discourse moving in the classroom and encouraging students to participate and engage. When used strategically and in combination with one another, they can help move the mathematical trajectory of the class forward. To illustrate that point, I analyze two transcript excerpts from a sixth-grade class described by Roy and colleagues (2017). The task discussed focuses on interpretation and application of the distributive property and is

Table 2 Teacher Discourse Moves can support productive and powerful math discourse (adapted from Herbel-Eisenmann et al. 2017).

Name of Move and Purpose	Examples
Waiting after a question or student response encourages broader participation, gives teachers the opportunity to consider what move to make next, holds students accountable for mathematical thinking, allows students to formulate new ideas, and provides students with processing time.	I would like you to think about this without offering an answer yet. Think about your response for a few minutes, and then I will ask you to turn to a neighbor to discuss it. Write down your initial ideas and the questions that you still have.
Inviting student participation encourages students to listen and respond, elicits diverse student perspectives, and positions students as authors of mathematical ideas and intellectual resources.	[Student], would you like to share your solution? What are your thoughts about this idea? Who would like to add on to [student's] response? Who has a question for [student]?
Revoicing allows a teacher to amplify, clarify, or extend a student idea; positions students as mathematical thinkers and doers; and sets up alignments and oppositions in a mathematical argument.	Let me try to say back what you said. . . . So what I am hearing is that you are saying _____, and [other student] is saying ______. Is that correct? It sounds like both [student 1 and student 2] are saying ______, which are similar ideas. Is that accurate?
Asking students to revoice serves similar purposes to teacher revoicing, positions students more strongly as mathematical thinkers and doers, and sets the stage for students to discuss their reasoning directly with one another.	Can someone else say that in his or her own words? Would someone repeat that idea using some of the math terms we have been using? Take a minute to think about the strategy that was just shared. Then turn to your partner and explain the idea as if that person were absent from class.
Probing a student's thinking can help a student reflect on and transform their thinking, support students in further articulating their mathematical ideas, make thinking more available to other students, and encourage shifts in students' mathematical language.	Can you explain to me why that works? Tell me about your solution. Can you tell me why you agree/disagree? Can you say more about your thinking using some of our mathematical terms?
Creating opportunities to engage with another's reasoning encourages students to use someone else's approach themselves, encourages active listening, and allows students to identify and explain similarities and differences among ideas, approaches, and explanations.	Choose one of the strategies that is not yours and solve and explain the problem using that strategy. In what situations do you think [student's] method would be most efficient? When would you not choose that method? How is [student's] explanation similar to and different from yours? This strategy represents an algebraic solution and this strategy uses a diagram. Which would you use for a future problem, and why would you make that choice?

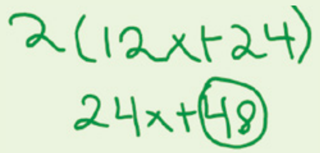
shown in **figure 2a** (see Roy et al. 2017 for a more detailed analysis of the mathematics). **Figures 2b–d** are student contributions made during the discussion. The excerpts and analysis that follow name the teacher discourse moves at play in the transcript and discuss how they serve to guide the mathematics and bring to the surface important mathematical thinking.

At the start of the discussion shown in **table 3**, the teacher *invites* Lynn's solution to be considered and requests that Nico explain the strategy. This *creates* opportunities for other students to engage with the mathematics. Through *probing*, the teacher gets students to explain the principles behind the strategy and assess its accuracy, and *revoices*

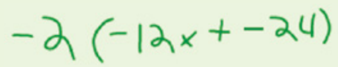
Fig. 2 The Distributive Property task (a) was examined by Lynn (b), Ali (c), and Kerry (d) (Roy et al. 2017, pp. 101–3).

Sofia thinks $2(12x + 24)$ can be rewritten as $6(4x + 8)$, but Andre thinks it cannot.
- Who is right? How do you know?
- What are other equivalent ways of writing $2(12x + 24)$?

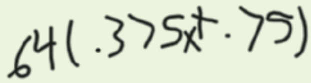
(a)



(b) Lynn's proposed expansion



(c) Ali's proposed equivalent expression



(d) Kerry's proposed decimal equivalent expansion

Table 3 Students explored one another's work in excerpt 1 from a sixth-grade distributive property lesson (Roy et al. 2017).

Speaker	Dialogue	Teacher Discourse Move
Teacher	Let's talk about it. . . . I am going to have you make sense of what Lynn is doing. [See **fig. 2b**.] Nico, tell us what you think she did.	Inviting student participation Creating opportunities to engage with another student's thinking
Nico	I think she multiplied 2 by 12 and 2 by 24.	
Teacher	Did she just multiply 2 by 12?	Probing student thinking
Student	She distributed the 2 to the 12 and 24.	
Pat	You do 2 times 12*x*.	
Shawn	I have a question. . . . Don't we need to do what's in the parentheses first before multiplying by 2?	
Teacher	Let's talk about that. Don't you need to do what's in the parentheses first? You know the rule that parentheses come first.	Revoicing
Student	I don't think. You can't do what is in parentheses first because you can't add 12*x* plus 24.	
Jamie	They are not like terms.	
…		
Ali	Let's see, $-2(-12x + -24)$. [See **fig. 2c**.]	
Teacher	We are going to have to stop for a second, because Cam got really excited.	Creating opportunities to engage with another student's reasoning
Cam	I did.	

a key question that relates directly to the goals of understanding the relationship between the distributive property and order of operations. In the next excerpt, the teacher *creates* opportunities to engage with other students' reasoning to discuss the novelty of the second strategy. The teacher discourse moves in this excerpt (*creating, probing, inviting*) are used to deepen student-to-student discourse and to go deep with the mathematics by eliciting student thinking. The discussion continues later on in the class period with the excerpt in **table 4**.

The second excerpt shows the teacher again asking students to make sense of a solution that is noted as unusual. By *probing* student thinking publicly, students talk to one another and add on to their explanation of the solution. This work of probing has the effect of clarifying explanations that may not be clear to all, as we see when the teacher invites Justice to help out, clarifying and connecting the explanations by Corey, Kerry, and Fran. By inviting Justice to share this group's number sense and reasoning, the numeric flexibility that was implicit in their calculations becomes visible to students. The impact on students shows when both Pat and Kerry offer similar strategies based on the powers of 2 and reciprocals, eliciting additional student thinking that was not evident. (A longer transcript can be found in **more4U**.)

In the excerpts, the teacher uses four of the six Teacher Discourse Moves. The longer transcript also features the teacher *asking* students to revoice (see **more4U**). *Waiting*, the sixth move, is not represented because the original recordings were not available to identify waiting instances. These moves individually and collectively bring important mathematical thinking to the surface about the distributive property task, promote students' collective mathematical argumentation as they re-explain and add to one another's ideas, and provide the teacher with formative assessment

Table 4 Excerpt 2 continues the sixth-grade distributive property lesson (Roy et al. 2017).

Speaker	Dialogue	Teacher Discourse Move
Teacher	Corey, tell us about this "crazy" one.	Creating opportunities to engage with another student's reasoning
Corey	It is not really one that people normally think of.	
Teacher	Why is that?	Probing student thinking
Corey	Because when most people, when they see decimal points, they think it is a lot harder than just whole numbers.	
Kerry	Once I saw a decimal number, I figured, yes, that could be right because 32 is bigger than 12 but if you multiply it by a decimal, it is going to get smaller; that would allow it to become 24.	
Fran	Thirty-two is bigger than 12 just like Kerry said we need to multiply it by a number less than 1 to make it smaller; three-fourths of 32 is 24. . . .	
Teacher	Justice, help him out. You said you do not need a calculator.	Inviting student participation
Justice	You really don't 'cause kind of what Fran said, 3 times 32 is 96 and then divide it by 4, which equals 24 plus 1 1/2 make that improper and then you multiply. . . .	
Pat	I think that you could do the same thing that you did with 0.75; you could do it an easier way. You know that 1 times 32, so split 32 in half and add half of 32 and 32 together. . . .	
Kerry	I have another one; 64 times (.375x + .75). [See **fig. 2d**.]	
Teacher	Which one of the other equivalent expressions is this like, Bailey?	Probing student thinking
Bailey	[32(.75x + 1.5)] They multiplied the 32 by 2 to get 64 and then divided the numbers inside the parentheses by 2.	

data related to the lesson goals. Resonant with the 5 Practices, the teacher has clearly *monitored* student thinking and sequenced student responses to share. The discussion begins with the teacher using the teacher discourse move of *creating*, which elicits ideas, followed by *probing* and *asking* students to revoice one another. These moves solicit more and deeper mathematical thinking. The *probing* and *revoicing* sharpens the mathematical language and bridges the verbal explanations and symbolic representations. The teacher also *invites* student participation, both generally and specifically, at key points in the lesson to add new mathematical perspectives and broaden participation. In using these moves, students have brought forward interesting mathematical ideas that are sharpened and honed to move the whole class toward the lesson goals. In so doing, students have been positioned as authors of mathematical ideas.

CONCLUSION

Facilitating meaningful mathematics discourse does not just happen by chance as teachers are engaged in teaching a mathematics lesson. Planning for meaningful discourse helps teachers ask strong questions, brings key mathematical ideas to the surface, and positions students as authors of mathematical ideas. In turn, this teaching provides richer data about what students know and understand relative to the lesson's mathematical goals. Tools like the 5 Practices and Teacher Discourse Moves can help break the cycle of Initiate-Respond-Evaluate and position students to engage with one another's mathematical thinking and reasoning. Using these tools across the planning, teaching, and assessing cycle will deepen and strengthen the mathematics discourse in your classroom.

Critical to these tools is the selection of tasks of high cognitive demand (Boston et al. 2017; Smith, Steele, and Raith 2017) that press students to make meaningful mathematical arguments and focus on the development of conceptual understanding. The *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith and Stein 2018) provide big-picture planning tools to chart a course through such tasks and support you as you teach with purposeful questions and supports to elicit and use student thinking. The Teacher

Discourse Moves found in *Mathematics Discourse in Secondary Classrooms* (Herbel-Eisenmann et al. 2017) are smaller-scale tools that when used thoughtfully can be critical scaffolds to small- and whole-group conversations. Using the discourse moves while enacting the 5 Practices helps to ensure broad participation, to sharpen the mathematical ideas and make them available to all students, and to position the discussion of a mathematical task as a meaningful opportunity for collective mathematical argumentation. Together, these two tools position teachers to place students' mathematical thinking at the center of our classroom practice.

ACKNOWLEDGMENT

I am indebted to George Roy, Sarah Bush, Thomas Hodges, and Farshid Safi for their thoughtful analysis of the sixth-grade class from their article and for allowing me to reanalyze the transcript for this article.

This material is based on work supported while serving at the National Science Foundation (NSF). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the reviews of the NSF.

BIBLIOGRAPHY

Boston, Melissa D., Fred Dillon, Margaret S. Smith, and Stephen Miller. 2017. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9–12*. Reston, VA: National Council of Teachers of Mathematics.

Carpenter, Thomas P., Megan Loef Franke, and Linda Levi. 2003. *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary Schools*. Portsmouth, NH: Heinemann.

Chapin, Suzanne H., Mary Catherine O'Connor, and Nancy Canavan Anderson. 2013. *Classroom Discussions: Using Math Talk to Help Students Learn, Grades K–6* (3rd ed.). Sausalito, CA: Math Solutions.

Herbel-Eisenmann, Beth A., Michelle Cirillo, Michael D. Steele, Samuel Otten, and Kate R. Johnson. 2017. *Mathematics Discourse in Secondary Classrooms: A Practice-Based Resource for Professional Learning*. Sausalito, CA: Math Solutions.

Lampert, Magdalene, and Paul Cobb. 2003. "Communication and Language." In *A Research Companion to Principles and Standards for School Mathematics*, edited by Jeremy Kilpatrick, W. Gary Martin, and Deborah Schifter, pp. 237–49. Reston, VA: National Council of Teachers of Mathematics.

Mehan, Hugh. 1979. *Learning Lessons: Social Organization in the Classroom*. Cambridge, MA: Harvard University Press.

Michaels, Sarah, Catherine O'Connor, and Lauren B. Resnick. 2008. "Deliberative Discourse Idealized and Realized: Accountable Talk in the Classroom and in Civic Life." *Studies in Philosophy and Education* 27, no. 4 (July): 283–97.

National Council of Teachers of Mathematics (NCTM). 2014. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Roy, George J., Sarah B. Bush, Thomas E. Hodges, and Farshid Safi. 2017. "Mathematics Discussions: Expectations Matter." *Mathematics Teaching in the Middle School* 23, no. 2 (October): 98–105.

Smith, Margaret S., Michael D. Steele, and Mary Lynn Raith. 2017. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6–8*. Reston, VA: National Council of Teachers of Mathematics.

Smith, Margaret S., and Mary Kay Stein. 2018. *5 Practices for Orchestrating Productive Mathematics Discussions* (2nd ed.). Reston, VA: National Council of Teachers of Mathematics.

Staples, Megan, and Sheryl King. 2017. "Eliciting, Supporting, and Guiding the Math: Three Key Functions of a Teacher's Role in Facilitating Meaningful Mathematical Discourse." In *Enhancing Classroom Practice with Research Behind Principles to Actions*, edited by Denise A. Spangler and Jeffrey J. Wanko, pp. 37–48. Reston, VA: National Council of Teachers of Mathematics.

Steele, Michael D., and Margaret S. Smith. 2018. "The Mathematical Tasks Framework and Formative Assessment." In *Eliciting and Using Evidence of Student Thinking to Guide Instruction: Linking Formative Assessment to Other Effective Instructional Practices*, edited by Edward A. Silver and Valerie L. Mills. Reston, VA: National Council of Teachers of Mathematics.

Michael D. Steele, steelem@uwm.edu, is a professor of mathematics education in the School of Education at the University of Wisconsin–Milwaukee. He directs the secondary mathematics teacher program and works extensively with practicing teachers to promote effective student-centered teaching practices. His research interests include studying teacher knowledge and practice, developing mathematical knowledge for teaching, and the role of discourse in student learning. He is currently on detail with the National Science Foundation as a program officer in the Division of Research on Learning.

more4U

A transcript is online at **https://www.nctm.org/mtms**. More4U items are available to NCTM members only.