INSIGHTS from the MAA National Study of College Calculus

Students who succeed in high school calculus become discouraged and quit in college. ***Why?***

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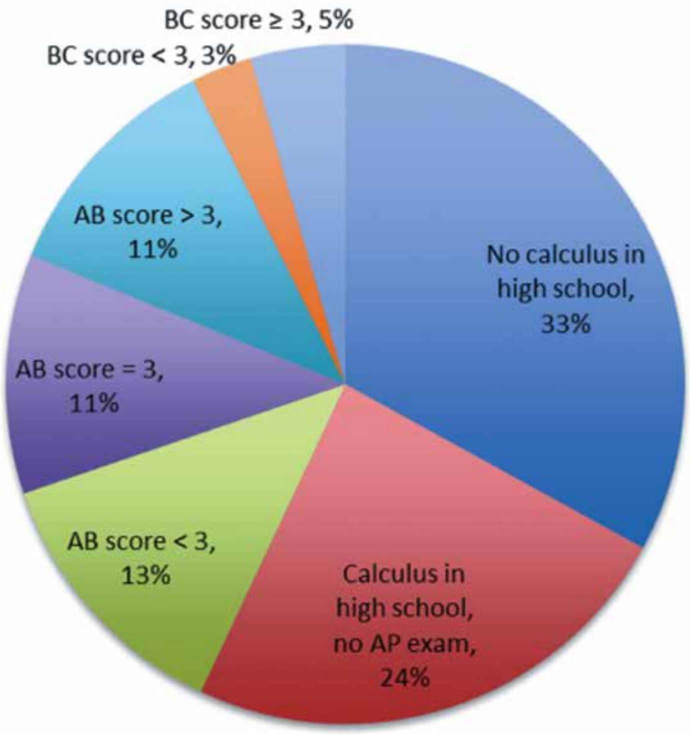
Over the past five years, the Mathematical Association of America, with support from the National Science Foundation, has explored the teaching of mainstream Calculus 1 at the postsecondary level, where by *mainstream* we mean those courses that can be used as part of the prerequisite stream to more advanced postsecondary mathematics. We surveyed 213 colleges and universities, 502 instructors, and more than 14,000 students to learn who takes Calculus 1 in college, why they take it, their preparation for this class, and their experience in this class. We also began to identify the characteristics of those classes that are most successful in encouraging students to continue their pursuit of mathematics. Following up on these surveys, teams of researchers visited twenty of these institutions, including community and technical colleges, liberal arts colleges, and public and private universities, to see firsthand what some of the best programs were doing. Here are some findings from this study, findings that should be of interest to those who are preparing students to succeed in college-level mathematics. A full account of the results of the study has been published in Bressoud, Mesa, and Rasmussen (2015); links to this report and research papers from the study are posted at www.maa.org/cspcc.



Note: AB and BC refer to scores on AP Calculus exams. The AB exam covers a little more than one semester of college calculus; the BC exam covers two full semesters. A score of 3 is roughly equivalent to a C at a major state university, a 4 to a B, and a 5 to an A.

Fig. 1 This chart shows the distribution of high school mathematics experience of students in Calculus 1 at research universities.

WHO TAKES CALCULUS?

In fall 2010, 300,000 students began the study of Calculus 1 at a college or university in the United States. By comparison, this past year approximately 750,000 students took a calculus course in high school. The result is that enrollment in Calculus 1 is dominated by students who have already seen this material. At the major state and private universities, which we refer to as *research universities* and which are characterized by offering a PhD in mathematics, two-thirds of the Calculus 1 students had completed a calculus course while in high school and more than a quarter had earned a 3 or higher on an AP® Calculus exam (see **fig. 1**). (For information on the AP Calculus exams, see the College Board (2012).) Students enrolled in Calculus 1 at these universities had been fast-tracked in high school; 77% had taken Algebra 2 by tenth grade (see **table 1**). They were among the best. Their high school mathematics GPA averaged 3.77, and their median SAT Math score was 670.

Although the results of our study will question the wisdom of the rush to calculus for many students, another perspective indicates a growing disparity between those who are able to take advantage of accelerated courses and those who are not. Only half of all high schools offer calculus (Office for Civil Rights 2014). Although black and Latino students make up 37% of all high school students, they constitute only 20% of the students who take the AB Calculus exam and only 11% of those who take the BC exam. The result is that when students from disadvantaged schools attempt a major that requires Calculus 1, they see themselves competing against students who have had a far richer experience in mathematics. As we explore the characteristics of typical Calculus 1 students, it will be important to keep in mind those who are not represented in these descriptions.

ARE THEY READY?

On entering Calculus 1, students expressed high levels of confidence in their mathematical abilities (more than 90% were confident), enjoyment of mathematics (more than 85% said that they enjoyed mathematics), understanding of the mathematics they had studied (almost 95%), and readiness for Calculus 1 (almost 95%) (see **fig. 2**). It is interesting that there is so little variation in these attitudes among students at different types of institutions, although students at undergraduate colleges do seem to enjoy mathematics slightly less than others. It is also interesting that students' sense of readiness for this course is even higher than their confidence in their mathematical abilities.

Table 1 Courses Taken in High School by Students in Calculus 1 at Research Universities

	Percentage of Calculus 1 Students	Mean Grade (Standard Deviation)
Algebra 2 at or before Tenth Grade	77%	3.8 (0.5)
Precalculus at or before Eleventh Grade	67%	3.7 (0.6)
Calculus in High School	67%	3.8 (0.5)
Statistics in High School	9%	3.6 (0.6)

Note: Percentages do not include students who studied an integrated curriculum.

These are students who are comfortable using graphing calculators; 88% reported being allowed to use them at least sometimes on exams in their last year of high school mathematics, although only 33% reported that they had always been allowed to use them on exams. Most students (86%) reported being at least somewhat comfortable doing complex calculations without the aid of a graphing calculator, but only 58% were completely comfortable doing these calculations by hand.

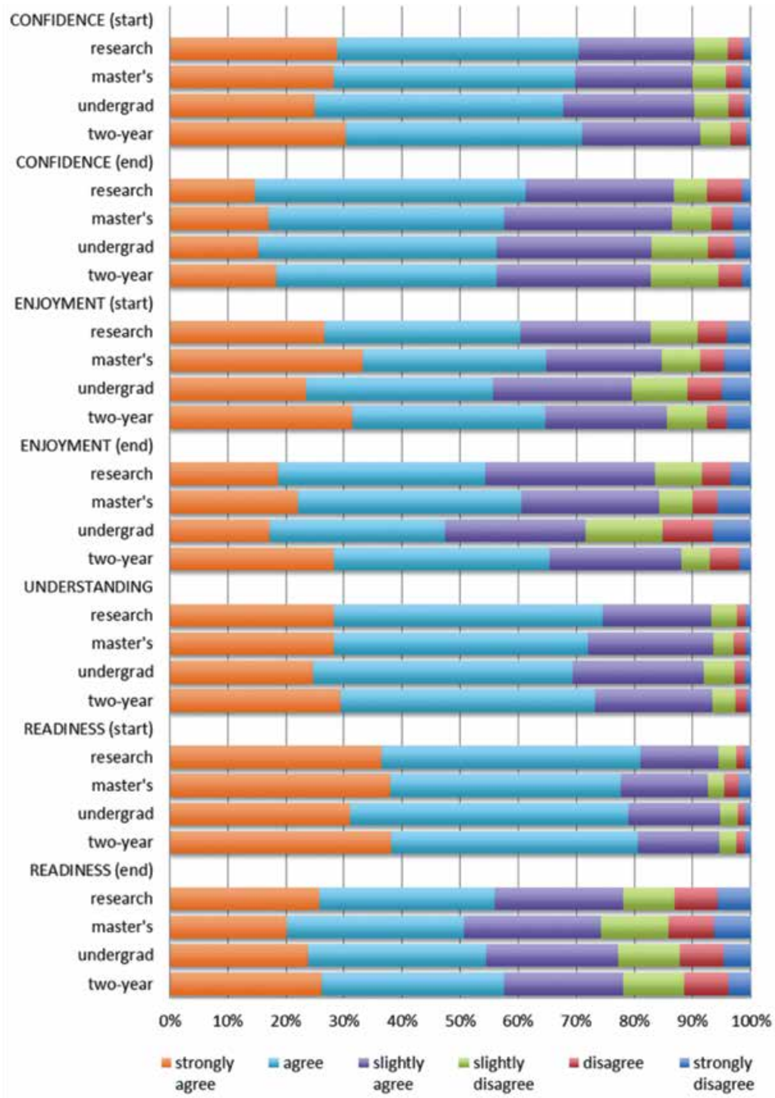
WHERE DOES CALCULUS LEAD?

Most students who enroll in Calculus 1 at any post-secondary institution are going into either the biological sciences (including health sciences) or engineering (see **fig. 3**). Only 6% are heading into the mathematical or physical sciences, a statistic that may be explained by the fact that many students heading into these fields qualify for placement directly into Calculus 2 or higher.

What was particularly interesting was the sharp difference between the career goals of men and women (see **fig. 4**). Of the men, 48% were going into engineering or computer science, whereas these majors attracted only 16% of the women. Women were heading predominantly for the biological sciences or teaching (53%), fields that together attracted only 23% of the men.

Although attitudes were similar across all types of institutions, socioeconomic status varied widely. Students at research universities were the most likely to have at least one parent who completed college and the least likely to be concerned about paying for college (see **fig. 5**). Students at undergraduate colleges also had highly educated parents and only slightly higher levels of concern about paying for college. Not surprisingly, students at the two-year colleges were least likely to have a parent who graduated from college and had the greatest concern about paying for their education, although even of these, 44% had a father who had completed college, 40% a mother with this level of education, and 22% had no concerns about paying for their education. Because universities for which the highest degree awarded in mathematics is a master's are predominantly public universities with a regional focus, it is not surprising that the socioeconomic status of their Calculus 1 students was closer to that of two-year colleges than of research universities.

Once they got to college, most Calculus 1 students spent roughly six to seven hours a week on calculus outside class; students at two-year colleges spent a bit more time, and students at four-year undergraduate colleges spent a bit less time. These study time data were collected at the end



Note: Agreement is indicated to the following statements at the start and end of the course:

Confidence: "I am confident in my mathematics abilities."
Enjoyment: "I enjoy doing mathematics."
Understanding: "I understand the mathematics that I have studied."
Readiness (*start*): "I believe I have the knowledge and abilities to succeed in this course."
Readiness (*end*): "My previous math courses prepared me to succeed in this course."

Fig. 2 Self-assessment of students in Calculus 1 at the start and the end of term is compared by type of institution, determined by highest degree given in mathematics.

Students' sense of readiness is even higher than their confidence in their mathematical abilities.

Table 2 End-of-Term Self-Reporting of Time Spent in Hours per Week, by Institution Type: Mean and Interquartile Range				
	Research Universities	Master's Universities	Undergraduate Colleges	Two-Year Colleges
Working at a Job	3.5 [0, 3]	9.5 [0, 18]	7.5 [0, 13]	12.5 [0, 25½]
Extracurricular Activities	6 [3, 8]	6 [0, 8]	10.5 [3, 18]	5 [0, 8]
Preparation for All Classes	18.5 [13, 25.5]	15.5 [8, 13]	16.5 [8, 25.5]	15 [8, 18]
Preparation for Calculus	6.5 [3, 8]	7 [3, 8]	5.5 [3, 8]	8.5 [3, 13]

Note: The top number is the mean. The first number in brackets is at the first quartile; the second number is at the third quartile. Data are heavily skewed. Full-time students accounted for 99% at research universities, 91% at master's universities, 98% at undergraduate colleges, and 76% at two-year colleges.

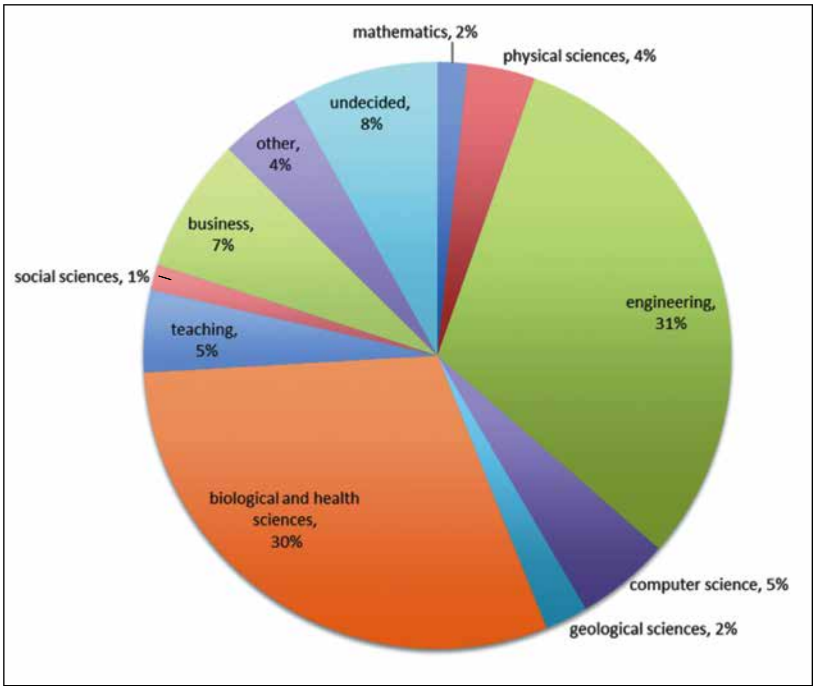


Fig. 3 Career goals vary for students in Calculus 1.

The teaching characteristics most strongly correlated with student desire to continue in mathematics are much more frequently found in the high school classroom than the university lecture hall.

of the term and thus represent only the students who were successfully completing this course. These students also reported that preparation for calculus constituted a third to a half of all their study time (see **table 2**). At two-year colleges, this figure is at least partially explained by the fact that a quarter of Calculus 1 students are part-time.

WHAT IS LOST?

The picture that emerges of the young people who enroll in Calculus 1 in U.S. colleges and universities is of students who are, for the most part, privileged, talented, and very confident. One of the clearest conclusions to come out of our study was how effective this course is in destroying that confidence.

At research universities and undergraduate colleges, just more than half of all Calculus 1 students earned an A or B, the grades that are the clearest signals that one is qualified to continue. At the master's universities and two-year colleges, only 43% received an A or B. Failure rates—D, F, or withdrew from the course (DFW)—were particularly high at the master's universities and two-year colleges (see **fig. 6**).

Having done well on an AP Calculus exam helped, but even among those students who had earned a 3 or higher, 18% of those who took Calculus 1 in college had received a D or an F or had withdrawn from the course and another 18% received a C (see **fig. 7**). We also found that those who had taken calculus in high school but earned less than 3 on the AP exam had a grade distribution in Calculus 1 that was comparable to that of the students who had not seen calculus in high school.

Between the start and the end of the students' college calculus class, their confidence and

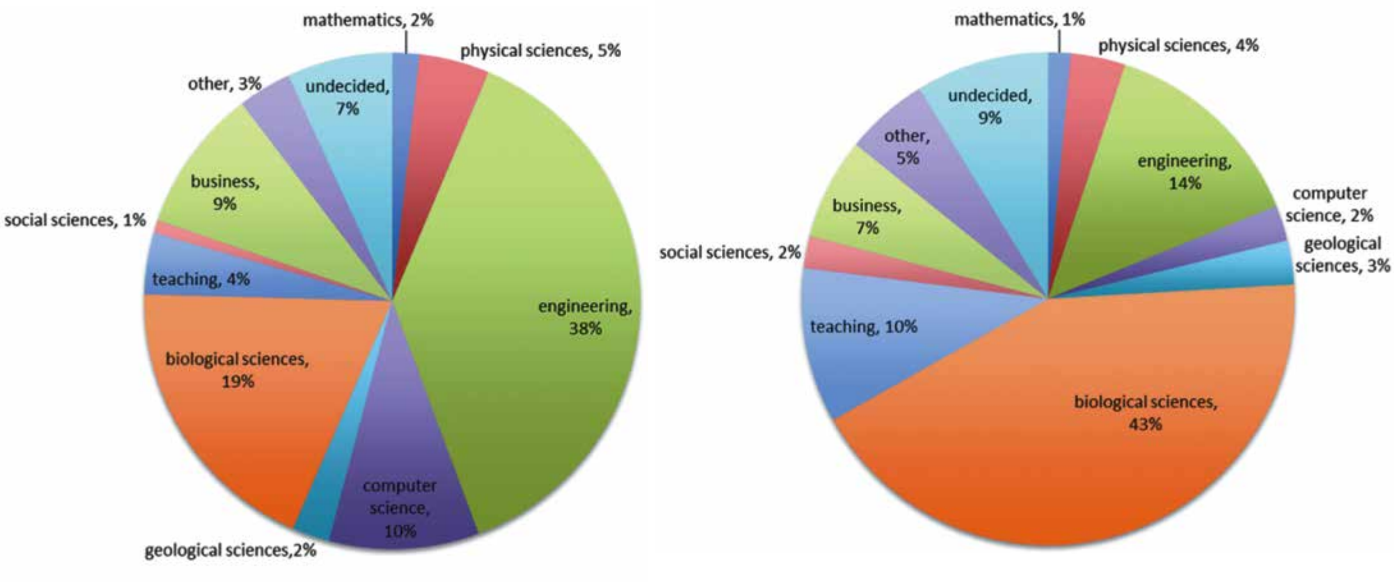
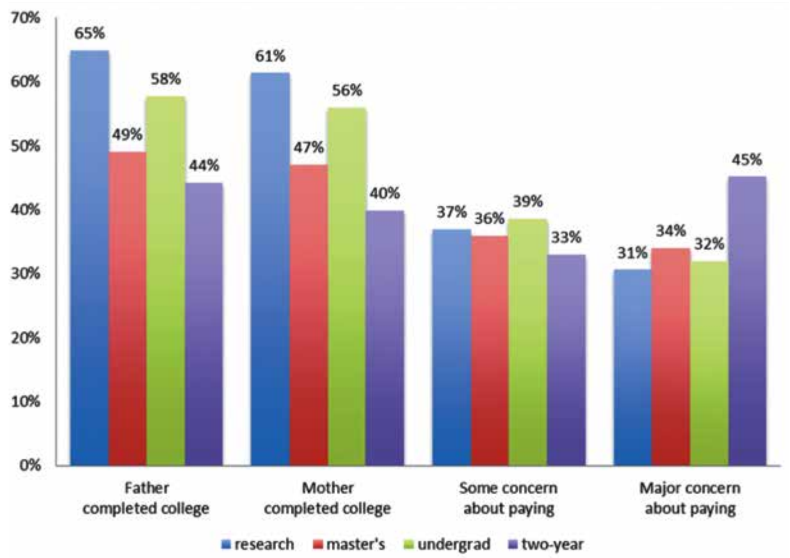


Fig. 4 Career goals vary for men (**a**) and women (**b**) in Calculus 1.

enjoyment of mathematics dropped sharply, with confidence falling by half a standard deviation and enjoyment of mathematics by a third (see **fig. 2**). Because the surveys were completed online outside class time, they reflect the opinions of the most responsible students. The distribution of final grades among those who completed the initial survey was consistent with that of the instructor-reported final grades, indicating that we had a representative sample of students of all ability levels. However, at the end of the term, the responses reflect the opinions of the successful students, 80% of whom were earning an A or B. This change in the distribution of respondents makes these shifts of attitude even more dramatic. Of particular interest is *readiness*. At the start of the term, it describes agreement with the statement "I believe I have the knowledge and abilities to succeed in this course." At the end of the term, it describes agreement with the statement "My previous math courses prepared me to succeed in this course." We see a drop from around 80% who agreed or strongly agreed at the start of the term to around 50% at the end of the term.

The drop in confidence had a particularly strong effect on women. Among the women who entered Calculus 1 with the intention of continuing on to Calculus 2, 10% of those who received an A, 13% of those who received a B, and 24% of those who received a C had changed their minds about continuing on to Calculus 2 by the end of that term. These rates were roughly twice as high as those for men.

We also found that women tended to be more self-critical of their understanding of calculus.



Note: *Some concern* indicates that the respondent slightly disagrees or slightly agrees with the statement "I am anticipating difficulty paying for college." *Major concern* indicates agreement or strong agreement with this statement.

Fig. 5 Indicators of socioeconomic status are compared by institution type.

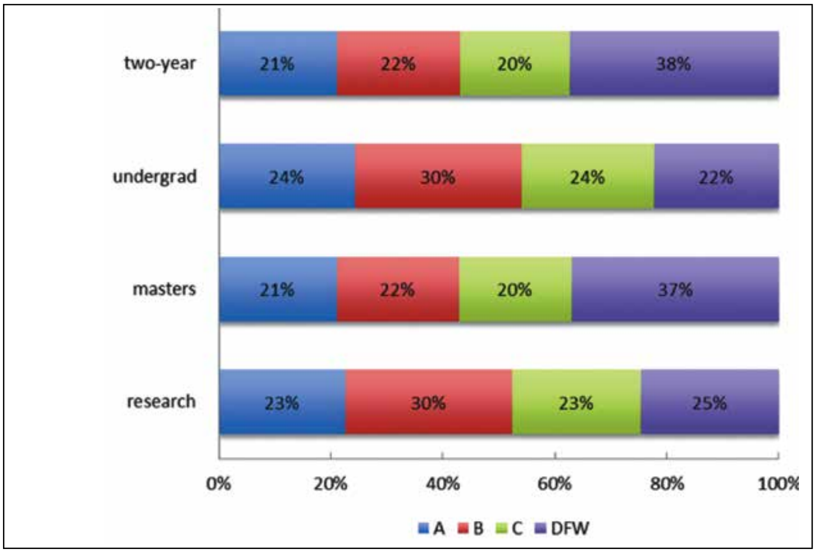


Fig. 6 Instructor-reported grades for Calculus 1 are compared by institution type.

We also found that women tended to be more self-critical of their understanding of calculus. Among those receiving an A or B for Calculus 1 who had changed their minds about continuing on to Calculus 2, 18% of the women but only 4% of the men gave as a reason that they did not understand calculus well enough. Even though they were earning an A or B for the course, 7% of the women gave as a reason for not continuing that their grade was not good enough. Not a single man earning an A or B gave this as a reason for discontinuing. We also found much lower rates of women among older and part-time students. These findings may be tied to the cultural message, strengthened by a paucity of role models, that only those women with "natural mathematical talent" belong in the most mathematically intensive disciplines.

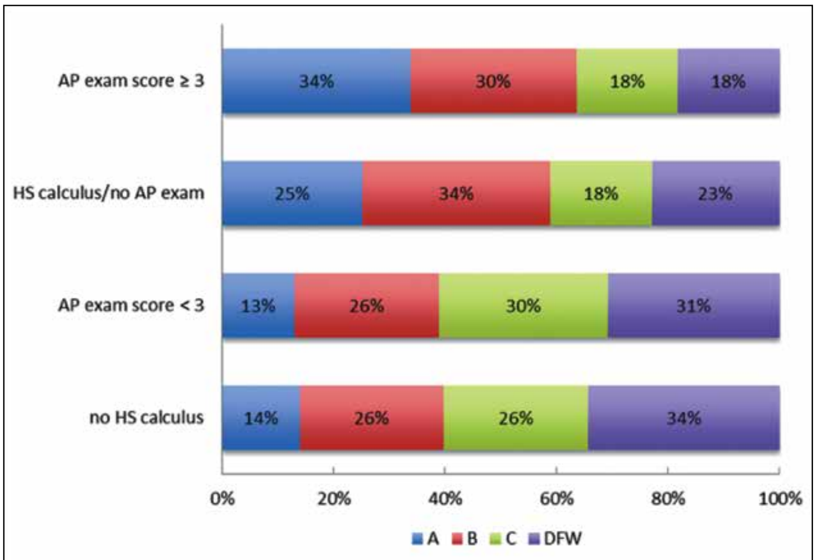


Fig. 7 Final grades in Calculus 1 are compared by mathematics experience in high school.

WHAT CAN BE DONE?

Why does this chasm between high school and college performance exist? Why is Calculus 1 in college so discouraging, even for those who do well in it? A few contributing factors are obvious, and our study has indicated others, but much still needs to be done to understand the dynamics of this transition.

Pacing is one consideration. An AB Calculus syllabus that is spread over 180 classes in high school is compressed into 45 or fewer class meetings in college. Class size and instructor qualification constitute another concern. Universities meet the challenge of teaching a thousand or more Calculus 1 students in a single term either by using their best instructors in very large classes or by opting for smaller classes where budget constraints often require hiring part-time or less experienced instructors, often graduate students whose preparation and support for teaching these classes may be minimal.

From our look at what happens inside the Calculus 1 class, the characteristics most strongly correlated with maintaining student confidence, enjoyment of mathematics, and desire to continue in mathematics were grouped under a heading we are calling *good teaching* (see **fig. 8**). These are characteristics that largely reflect the rapport between student and instructor and are much more frequently found in the high school classroom than the university lecture hall.

We also found that active learning approaches, what we are calling *ambitious teaching*, helped (see **fig. 9**) but with a caveat: The strongest gains came only if the instructor also rated high in good teaching. Without good teacher-student rapport, these ambitious teaching strategies could be counterproductive.

WHAT ARE THE LESSONS FOR HIGH SCHOOL TEACHERS?

First, just because students can succeed in calculus in the supportive environment of a high school does not guarantee that they will be successful when they get to college. The most useful skill for success in college is the ability to learn on one's own, to be able to think critically about what one reads or views in videos, and to use this critical analysis to build a personal, coherent, and functional mental structure for the many concepts of calculus. Getting students successfully through a test, even an AP Calculus exam, is by itself no guarantee that they will be successful in college.

Second, a calculus experience in high school that does not rise to the level of being able to earn a 3 or higher on an AP Calculus exam does not seem to have any effect, positive or negative, on

Good Teaching: Top Ten Student-Reported Instructor Practices

1. My calculus instructor provided explanations that were understandable.
2. My calculus instructor listened carefully to my questions and comments.
3. My calculus instructor helped me become a better problem solver.
4. My calculus instructor allowed time for me to understand difficult ideas.
5. My calculus instructor made me feel comfortable in asking questions during class.
6. My calculus instructor presented more than one method for solving problems.
7. My calculus instructor made class interesting.
8. My calculus instructor asked questions to determine whether I understood what was being discussed.
9. My calculus exams were a good assessment of what I learned.
10. My calculus instructor discussed applications of calculus.

Fig. 8 Classroom practices characterize good teaching.

Ambitious Teaching: Top Ten Student-Reported Instructor Practices

1. My instructor frequently had students work with one another.
2. Assignments completed outside class time were frequently submitted as a group project.
3. The exam questions required that I solve word problems.
4. The assignments completed outside class time required that I solve word problems.
5. My instructor frequently required me to explain my thinking on my homework.
6. The assignments completed outside class time required that I solve problems unlike those done in class or in the book.
7. My instructor frequently asked students to explain their thinking.
8. My instructor frequently held whole-class discussion.
9. The exam questions required that I solve problems unlike those done in class or in the book.

Fig. 9 To be effective, ambitious teaching strategies require strong student-teacher rapport.

performance in Calculus 1 when the student gets to college. This finding draws into question much of the rush to calculus in high school. It has become a cliché in U.S. colleges and universities that students fail calculus not because they do not understand calculus; they fail it because they have not mastered precalculus. An alternative to calculus in high school that focuses on strengthening students' understanding of algebra, geometry, trigonometry, and functional relations while building problem solving skills would be very welcome.

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