Journal for Research in Mathematics Education
2018, Vol. 49, No. 3, 292–329

The Path to College Calculus: The Impact of High School Mathematics Coursework

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This study addresses a longstanding question among high school mathematics teachers and college mathematics professors: Which is the best preparation for college calculus—(a) a high level of mastery of mathematics considered preparatory for calculus (algebra, geometry, precalculus) or (b) taking calculus itself in high school? We used a data set of 6,207 students of 216 professors at 133 randomly selected U.S. colleges and universities, and hierarchical models controlled for differences in demography and background. Mastery of the mathematics considered preparatory for calculus was found to have more than double the impact of taking a high school calculus course on students' later performance in college calculus, on average. However, students with weaker mathematics preparation gained the most from taking high school calculus.

Keywords: Advanced Placement; College calculus; High school calculus; High school to college transition; Mathematics teaching

"*Stop wasting time on calculus in high school! Concentrate instead on making sure your graduating students have a rock-solid foundation in algebra and a good understanding of logarithms, exponentials, and trigonometric functions. Rushing on to teach calculus to students who are not sure about the rules for adding fractions does nobody any good.*"—College calculus professor

"*I think that you will find that 'having previously taken calculus while in high school' (regardless of the high school grade received) will be a stronger predictor of doing well in College Calculus than any activity, intervention, or reform.*" —High school calculus teacher

Without the excellent contributions of many people, the Factors Influencing College Success in Mathematics, or FICSMath, project would not have been possible. We thank the members of the FICSMath team: John Almarode, Devasmita Chakraverty, Jennifer Cribbs, Kate Dabney, Zahra Hazari, Heather Hill, Jaimie Miller, Matthew Moynihan, Jon Star, Robert Tai, Terry Tivnan, Annette Trenga, Carol Wade, and Charity Watson. We would also like to thank several mathematics educators who provided advice or counsel on this project: Sadie Bragg, David Bressoud, James S. Dietz, Joan Ferrini-Mundy, Solomon Garfunkel, Daniel Goroff, Ed Joyce, Carl LaCombe, James Lewis, Karen Marrongelle, William McCallum, Ricardo Nemirovsky, and Bob Speiser. Last but not least, we are grateful to the many college calculus professors and their students who gave up a portion of a class to provide data. This work was supported by the National Science Foundation under Grant No. 0813702. Any opinions, findings, and conclusions in this article are the authors' and do not necessarily reflect the views of the National Science Foundation. The authors have no financial interest or benefit arising from the direct applications of their research.

There exists a wide gulf between the views of high school teachers and college professors concerning student preparation for college calculus (Stroumbakis, 2010; Wade, Sonnert, Sadler, Hazari, & Watson, 2016). High school mathematics teachers predictably view their courses as valuable preparation for further study in college mathematics, yet many college professors have expressed doubts about their worth, with fewer than 15% believing that college students are well prepared for college-level work in mathematics (Ferrini-Mundy & Gaudard, 1992; Mooney, 1994; Sanoff, 2006). Many high school teachers report that their graduates tell them of the usefulness of their high school preparation. However, the actual number of graduates who return and share such stories is small, and few students return to say that their high school course was not helpful, an example of serious sampling bias.

College calculus is arguably the culmination of years of a sequenced and cumulative precollege mathematics preparation (Ryals, 2014). Calculus is also deemed the mathematical gateway to a career in science, technology, engineering, and mathematics (STEM) as well as in other fields (e.g., medicine; Gainen, 1995; Lovecchio & Dundes, 2002; Seymour & Hewitt, 1997). Among students entering college, 35% of males and 20% of females intend to pursue a STEM career (National Science Board, 2014). This makes college calculus a strategic research site. Yet, national data are scant on the path that students follow on their way to college calculus. Of particular importance may be the choice to take a calculus course in high school, which to a considerable degree, depends on when students first take Algebra I (Jaffe, 2012; Moses & Cobb, 2001). A large fraction of students in the United States begin high school mathematics with Algebra I in ninth grade and then follow a sequence of geometry and Algebra II that ends with precalculus in 12th grade. These students can then go on to take calculus for the first time in college. Many others begin this same sequence in eighth grade and end with precalculus in 11th grade, opening several possibilities for study in their senior year. These include a "year off" from mathematics, taking statistics or other mathematics courses offered (e.g., finite mathematics, integrated math), or taking calculus.

Over the last decade, the variety of mathematics and science courses offered in high school has proliferated. Enrollment in Advanced Placement® (AP) calculus courses has increased by 54% during the last decade, far exceeding the 6% increase in the number of students graduating from high school (College Board, 2017; Hussar & Bailey, 2017). Although in all but elite colleges and universities, a high score on an AP exam (i.e., 3, 4, or 5) can earn a college semester's credit for calculus, many of these accomplished students opt to retake calculus in college, often following the advice of their college advisor or others already majoring in STEM fields (Bressoud, 2010a). Nearly all STEM majors require at least one term of college calculus (Bressoud, Mesa, & Rasmussen, 2015). Those who are interested in a medical career may be confused by "the cacophony of medical school dictates" (Nusbaum, 2006, p. 166) concerning calculus coursework, but, currently, 30% of American and Canadian medical schools (48 out of 162) require or recommend it (Association of American Medical Colleges, n.d.).

Bressoud's (2010b) study carried out by the Mathematical Association of America also holds potential lessons for college mathematics professors who set policy for how AP exam scores can be used for the awarding of college credit, for the satisfaction of requirements, or for placement into higher level courses (Bressoud, 2007; Committee on the Undergraduate Program in Mathematics Panel, 1987; Ferrini-Mundy & Gaudard, 1992). Bressoud, Carlson, Mesa, and Rasmussen (2013) found that 16% of students enrolled in introductory college calculus in the United States had earned a score of 3 or higher on their AP calculus exam in high school. Such students often experience an easier time in college calculus than those who did not take high school calculus, spending less time outside of class on assignments and studying while also reporting less boredom in class (Gibson, 2013). It remains an open question whether those who have taken AP calculus attain the course grade claimed as equivalent by the College Board's AP program based on their AP exam score (i.e., a score of 5 = "A," a score of 4 = "A-" to "B," a score of 3 = "B-" to "C"; Shaw, Marini, & Mattern, 2013).

College professors, high school mathematics teachers, mathematics education researchers, and students themselves all have opinions concerning how students are best prepared for success in college mathematics. This study presents empirical evidence that addresses these stakeholders' beliefs about the value of preparatory high school coursework in mathematics for students who later enroll in college calculus. We begin by outlining two theoretical perspectives and reviewing the literature concerning precollege preparation for college calculus. Drawing on data from a large national study of college calculus students, we trace the path of courses that students take to prepare for calculus. By combining performance in these courses with standardized test results for each student, we construct a composite encompassing knowledge of mathematics considered prerequisite for success in college calculus. Using a hierarchical linear model (HLM) to account for college, instructor, and student differences, we estimate the relative impact on college calculus performance of mastery of precalculus-level preparation versus taking a calculus course in high school. We then draw connections to the research literature and to the two theoretical perspectives. Finally, we discuss the implications of these findings.

In this study, we examine evidence that answers the following research question: What is the relative impact of mastery of mathematical prerequisites for calculus versus an exposure to calculus in high school on students' college calculus performance, controlling for students' background?

Theoretical Perspectives and Associated Research Literature

Of all the subjects taught at the precollege level, mathematics is arguably the most hierarchically organized, with the most historically stable subject matter (McFarland, 2006; Riegle-Crumb, 2006). However, precollege mathematics is beset by two conflicting theories related to the sequencing of the learning of highly structured subjects: Robert Gagné's (1968) theory of hierarchical learning and Jerome Bruner's (1960) spiral learning approach. Whereas Gagné holds that learning is maximized through the identification and mastery of prerequisite skills and concepts, Bruner downplays mastery, claiming that a more cyclical approach to curriculum optimizes learning. Whereas Gagné might argue for a full mastery of mathematics considered prerequisite to calculus before students are allowed to take calculus (hence, only needing to take introductory calculus once), Bruner's theory supports the idea that a more efficient process is for students to be exposed to calculus multiple times at increasingly sophisticated levels (hence, supporting taking introductory calculus in both high school and college).

In this section, we discuss prior research concerning those students who do succeed in college mathematics and, in particular, introductory college calculus. The peer-reviewed literature contains numerous studies that have investigated the predictive value of student demographic and mathematical background variables, particularly standardized test scores, high school grades, and high school course-taking choices, as predictors (Kaufman, 1990). Beyond the decisions of individual students, the structure of the precollege mathematics curriculum determines the sequence of students' exposure to mathematical concepts and the degree to which mastery of prerequisite mathematics determines calculus exposure in high school. Attention to prerequisites reoccurs in college because college placement exams in mathematics and AP exam scores govern the timing of access to college calculus. We pay particular attention to whether the studies cited support or diminish each of the two theoretical perspectives.

Hierarchical Approach to Preparation for Calculus

Building on Gagné's theory of hierarchical learning, Jones and Russell (1979) are considerably more stringent, claiming that without the mastery of prerequisite knowledge and skills, higher levels of knowledge cannot be attained. Others have reinforced this radical view of the difficulty of building advanced mathematical knowledge without mastering prerequisites; for example, Sfard and Linchevski (1994) have stated that "mathematics is a hierarchical structure in which some strata cannot be built before another has been completed" (p. 91). From this premise follows that without mastery of the prerequisites to calculus, students will fail (or at least face great difficulty) in learning college calculus. This view supports an approach that students lacking such mastery would be better served by delaying calculus enrollment until they are better prepared for it.

A belief in the necessity of a sequential structure is instantiated in the Algebra I–geometry–Algebra II–precalculus course sequence, which is designed to prepare students for calculus in high school or college. (Only recently have there been inroads of the multiyear Integrated Mathematics sequence.) Based on a sequential configuration, a strategy to improve readiness for college mathematics has evolved that requires an earlier start and more coursework. U.S. schools have seen a dramatic rise in the percentage of students taking higher level mathematics courses in high school (National Science Board, 2006, 2014; Planty, Provasnik, & Daniel, 2007; Snyder & Dillow, 2015; Snyder, Dillow & Hoffman, 2007; Wirt et al., 2002), and 12 states now require at least four courses in mathematics for graduation from high school (Stillman & Blank, 2009). Since 1990, enrollment in high school analysis and precalculus has tripled to 36%, and enrollment in high school calculus has more than doubled to 17% in 2009. The percentage of all high school students taking AP calculus exams has quadrupled from 1991 to 2017, growing from 0.7% of high school students in 1991 to 1.9% in 2009 and 2.8% in 2017 (College Board, 2002, 2006, 2017; Hussar & Bailey, 2017).

Course sequences are a particularly relevant example of hierarchical learning (Stevenson, Schiller, & Schneider, 1994), embodying the predictable product of a series of staged prerequisites that allow but a few paths through precollege mathematics (Schneider, Swanson, & Riegle-Crumb 1997; Stevenson et al., 1994). The trajectory that students experience on their chosen path through high school mathematics may be sequential, but it is also dynamic in that students may face obstacles along the way (Confrey, Maloney, Nguyen, Mojica, & Myers, 2009), with implications for success in subsequent courses. According to Reamer, Ivy, Vila-Parrish, and Young (2015), "Many students fail to master requisite concepts before advancing to more complex ideas, leaving them ill-prepared to succeed in higher level Science, Technology, Engineering, and Mathematics (STEM) coursework" (p. 4). The canonical sequence of mathematics courses in U.S. high schools starts with prealgebra; progresses through Algebra I, geometry, Algebra II, and trigonometry or precalculus; and culminates in calculus (Riegle-Crumb, 2006; Schiller & Hunt, 2011). This sequencing of courses reflects an institutionalized pedagogical understanding of how mastery of less complex topics is required before students can progress to more advanced topics (National Mathematics Advisory Panel, 2008). Students typically gain positional advantages from being placed in more advanced courses earlier in their schooling (Kelly, 2009; Schneider et al., 1997; Smith, 1996). Those who believe in the strict hierarchical nature of learning mathematics can argue that the major impediment to doing well in calculus is poor preparation. Rather than waste their time taking calculus in high school and again in college, students with weaker backgrounds need a course that strengthens their algebra and precalculus skills (in high school or college). Correspondingly, well-prepared students should perform strongly in a rigorous high school calculus course (e.g., AP calculus) and have no need to take it a second time, easily progressing to more advanced mathematics courses in college.

Evidence supporting the hierarchical approach includes student success on the AP calculus exams designed to emulate those created by professors teaching introductory college calculus. Data for 2016 show that 183,486 students (59.5%) taking the AP Calculus AB exam and 101,264 students (81.1%) taking the more rigorous AP Calculus BC exam earned passing grades (College Board, 2016). Mathematics professors have reported that although many students enter college calculus with prior exposure to elementary calculus, their lack of mastery of algebra is problematic (Bressoud, 2007; Ferrini-Mundy & Gaudard, 1992). This opens the question as to whether such students would be better served studying the mathematical prerequisites to calculus for a longer period of time in high school instead of taking a high school calculus course. Keng and Dodd (2008) found that AP calculus students who did not pass the AP exam fared less well in college calculus at the University of Texas, Austin than did students who had not taken AP calculus at all. Wilhite, Windham, and Munday (1998) found no statistically significant effect of taking high school calculus on college calculus grades of 182 students at the University of Arkansas at Fayetteville. However, they did find that ACT® test scores (a measure of mastery of mathematics prerequisite to calculus) were a statistically significant predictor. In a study of placement into calculus courses at 50 colleges, using an ACT mathematics score of 27 as a cut-off to show mastery of prerequisites raised the expected number of students earning a grade of "B" or better from 29% to 75% compared with not using any cut-off for entry (ACT, 2014). Likewise, the SAT mathematics portion does not test beyond introductory algebra and geometry.

Evidence opposing the hierarchical approach includes the worrisome finding that despite increases in the number of mathematics courses taken, high school graduates show no stronger mathematics comprehension or skill levels on the nationally representative National Assessment of Educational Progress (NAEP; National Center for Education Statistics [NCES], 2013). Twelfth-grade scores (testing algebra, geometry, and trigonometry) were flat from 1990 to 2012, including those of top performers.[1] Many mathematics professors also express skepticism concerning the quality of students' high school preparation for college calculus (Agustin & Agustin, 2009; Wade et al., 2016). Even students who seemingly have a strong high school mathematics preparation may find themselves placed in lower level college mathematics courses. Lichten (2000) reported that 17% of students passing their AP calculus exam (i.e., with a score of 3, 4, or 5) ended up in a developmental mathematics course (i.e., precalculus or college algebra) to fulfill requirements in the 14 colleges studied. Mastery of calculus may not necessarily imply a mastery of the mathematics considered to be preparatory for studying it. Some point to alternative explanations for this lack of improvement, particularly college mathematics placement exams being poorly designed to gauge calculus readiness (Fitchett, King, & Champion, 2011).

Spiral Approach to Preparation for Calculus

The alternative theory in the mathematics community concerning mastery of prerequisite knowledge holds that spiraling towards understanding is a more natural and efficient approach to learning than the sequential mastery of a series of concepts and skills believed to be essential. Jerome Bruner (1960) advocated this position at the National Research Council's seminal Woods Hole Conference in 1959, which was attended by 35 scientists and mathematicians intent on reinventing precollege STEM education in the United States. Bruner told the audience, "A curriculum as it develops should revisit these basic ideas repeatedly, building

[1] An alternative explanation is that by age 17, students might have learned that their performance on this test has no bearing on their future, so they may not be as motivated as younger students to perform well (Brophy & Ames, 2005).

upon them until the student has grasped the full formal apparatus that goes with them… the 'spiral curriculum' that turns back on itself at higher levels…" (p. 13). He continued to develop the idea of how precollege mathematics instruction might look: "Instruction in these subjects should begin as intellectually honestly and as early as possible in a manner consistent with the child's forms of thought. Let the topics be developed and redeveloped in later grades" (pp. 53–54).

Constructivists have built on this idea that students should encounter what are often considered advanced mathematical concepts in multiple environments and at multiple times early in their precollege years (Noble, Nemirovsky, Wright, & Tierney, 2001; Roschelle & Kaput, 1996; Tchoshanov, Blake, & Duval, 2002). For example, Dienes (1964) has advocated the qualitative exploration of mathematics through activities (e.g., physical manipulation, whole-body modeling), attributing great value to images and stories instead of just symbolic representations. He also advocated getting to calculus concepts early, using graphical approaches to the mathematics of change in middle school, and then learning the algebraic notation for the graphical concepts only later. The spiral approach can also foster an environment in which algebra, geometry, and precalculus skills can be further mastered in high school calculus because calculus "provide[s] a means of reviewing algebra and precalculus concepts" (Wade et al., 2016, p. 13).

Evidence supporting the spiral approach includes the finding that students can learn the mechanics of algebraic manipulation rules early but only come to learn the meaning of algebraic representations later (White & Mitchelmore, 1996). This is illustrated by Wagner's (1981) observation that many mathematics students first view two equations with only different letters for the variables as being completely different representations, and only later do they recognize their similarity. High school calculus can provide an opportunity to move from a superficial understanding of algebra as rules governing the manipulation of symbols to one where meaning is derived from connecting concepts to mathematical objects at lower levels of abstraction (White & Mitchelmore, 1996). In Betebenner's (2001) study, data from the Longitudinal Study of American Youth (LSAY) were employed to show that students are well prepared only for repeating their last high school course anew in college—in other words, a high grade in high school algebra is adequate preparation only for college algebra, not for college precalculus. Taking a calculus course in high school and again in college is an example of the spiral approach in that an introduction to some concepts is thought by many to later facilitate the learning of calculus later at a more rigorous level. Consistent with Betebenner's findings, the majority of students who enroll in college calculus have had a prior course in calculus in high school (Bressoud, 2015; Bressoud, Carlson, Mesa, & Rasmussen, 2013; Burton, 1989). Burton (1989) found that any exposure to calculus, even an abbreviated one, bestows benefits on college calculus students. Ferrini-Mundy and Gaudard (1992) found that taking high school calculus increased performance by nearly one letter grade (9.3 points on a 100-point scale) after controlling for SAT scores. Similarly, Fayowski, Hyndman, and MacMillan (2009) found that among 140 college calculus students, those with high school calculus (46% of the total) earned one letter grade higher (10.3 points on a 100-point scale). In the same way, mastery of more basic mathematical concepts and skills (e.g., functions, infinite series) may be acquired over several years of schooling. Treatment several times could give students a deeper understanding of their complexities and applications.

Evidence opposing the spiral approach includes research examining the effect of taking precalculus in college. Students who must take college calculus to fulfill requirements for their major cannot do so immediately if they are shunted to a college precalculus course. These courses cannot be considered particularly effective in maintaining student interest and motivation because only 40% of the students who enroll in and pass college precalculus courses ever go on to beginning college calculus (Sonnert & Sadler, 2014). Using discontinuity regression to gauge the impact of a college course in precalculus, Sonnert and Sadler (2014) found that a college precalculus course had no discernable influence on students' subsequent grade in college calculus. Spiraling though precalculus again was not effective for these students. Some argue that anything less than a year of AP calculus in high school is inadequate and that students should wait until college to take a rigorous calculus course (Wilhite, Windham, & Munday 1998). Lucas and Spivey (2011) warned that high school calculus may imbue many misconceptions that must be unlearned later.

Other Relevant Research

The lack of growth in NAEP scores coupled with the growth of remedial and developmental mathematics at the college level cast doubt on the efficacy of precollege mathematics courses (Rasmussen et al., 2011). Although many students do come to college prepared for college-level mathematics courses, a large fraction appear to have learned less than they need to. This difference in mathematics preparation has been attributed to a whole host of factors. Some focus on differences in natural mathematical abilities (Feigenson, Libertus, & Halberda, 2013); others focus on psychological constructs, such as mathematics anxiety (Aiken, 1970; Greene, Debacker, Ravindran, & Krows, 1999; Olsen & House, 1997; Thorndike-Christ, 1991), self-efficacy (Hackett & Betz, 1989), effectance motivation (Bretscher, Dwinell, Hey, & Higbee, 1989), students' perceptions of their teachers' expectations (Mandeville & Kennedy, 1993; Thomson & DeLeonibus, 1978), and "mathematics identity" (Cribbs, Hazari, Sonnert, & Sadler, 2015.). Still others focus on social factors, such as parental socioeconomic status, neighborhood, gender, and race (Oakes, 1990; Reyes & Stanic, 1988; Watson, 2012). Whereas these deep-seated factors may all play a role in shaping mathematics preparation as well as performance in college calculus, we have taken a more concrete approach, tracing students' mathematics trajectories through the sequence of high school mathematics classes and students' performance in them. These trajectories are open for modification by educators and school systems (in contrast to many of the factors above that are difficult or impossible for them to influence), which makes them interesting from a policy point of view. Moreover, they have been shown by research to predict subsequent mathematics outcomes.

Betebenner's work (2001) found that the most important factor in college mathematics success is the amount of mathematics coursework undertaken in high school. Taking Algebra I in eighth grade marks the start of the progression that typically results in students taking advanced mathematics courses in high school, and this pattern was found to be related to the educational level and involvement of their parents (Horn & Nuñez, 2000; Moses & Cobb, 2001). Students appear to rely primarily on their mothers' and teachers' advice for when to start algebra and how long to persevere in mathematics (Lutzer, Maxwell, & Rodi, 2002; Lutzer, Rodi, Kirkman, & Maxwell, 2007). High school mathematics course taking has also been found to have a remarkable effect even beyond college mathematics. For example, taking high school calculus had a much greater impact on college physics grades than did taking high school physics (Sadler & Tai, 2001; Tai, Sadler, & Loehr, 2005). This "calculus effect" also holds in college chemistry and biology (Loehr, Almarode, Tai, & Sadler, 2012; Tai, Sadler, & Mintzes, 2006).

On a systemic level, many fear that an AP calculus course passed in high school may be the last mathematics course that a student ever takes (Bressoud, 2010b; National Research Council, 2002, p. 59; Rasmussen et al., 2011). Lichten (2000) found that 39% of students who passed their AP calculus exams in high school (i.e., an AP exam score of 3, 4, or 5) did not later enroll in any level of college calculus.

Whereas at one time college calculus students shared a common history of the traditional precollege sequence of mathematics coursework (Algebra I, geometry, Algebra II, precalculus), many more options are currently available to high school students. Studies that have explored how students' high school course-taking decisions impact their performance in college mathematics courses have typically considered too few control variables that might also play a role. Particularly, the impact of demographic differences, parents' education, gender, race, ethnicity, and student career goals is often left unexplored. Frequently carried out within a single institution or for a single professor, samples are relatively small and not nationally representative, making it questionable whether findings can be generalized to the wider population. This study aims to ameliorate these limitations and to explore the impact of high school course taking and performance in mathematics while controlling for students' background in a national sample of students taking college calculus.

Data and Methods

The data for this article were drawn from the Factors Influencing College Success in Mathematics (FICSMath) study, an NSF-funded research project examining predictors of student calculus performance in college based on their precollege experiences. This large-scale study surveyed a nationally representative sample of 10,437 students drawn in the fall semester of 2009 from 336 professors' courses in a stratified random sample of 134 U.S. colleges and universities.

The methodological approach used differs from the hypothetical experimental ideal of randomly assigning students to take or not take high school calculus, which would be an unrealistic technique. Instead, we first examined the available data set to characterize the high school mathematics courses that students took on their way to college calculus. Controlling for student differences, we then used regression to examine how the degree of preparation for calculus and a student's decision to take calculus in high school affected their performance in college calculus. We also considered the interactions of these two variables of interest with the other independent variables and between themselves.

Survey

The FICSMath data set was collected using a six-page, 61-item survey to gather data about college calculus students' academic performance, course taking, and other experiences while in high school as well as their demographic background. Survey questions originated either from the research literature on the preparation for college mathematics or from online surveys of college professors, high school teachers, and college calculus students. The goal of the project was to be inclusive and test as many common stakeholders' hypotheses about the predictors of success as possible. During pilot administration, we made sure that scales and options accurately reflected student experiences. Careful attention was paid to demographic measures that predict college performance, noting prior research findings concerning students' gender, age, ethnic and racial background, and socioeconomic status.

To gauge reliability, we conducted a separate test–retest study in which 174 students from three different colleges took the survey twice, 2 weeks apart. For groups of 100, less than a 0.04% chance of reversal existed between the 50th and 75th percentiles (Thorndike, 1997, p. 117). It is important to recognize that self-reporting of such data has been widely studied among college students and is considered an accurate representation of coursework and grades (Sanchez & Buddin, 2015), especially when the survey questions are relevant to the subject (Kuncel, Credé, & Thomas, 2005). For students in their first few weeks of college calculus, one can imagine that there is little that is more relevant to them than their own degree of preparation for the course in which they are enrolled.

Sample

The FICSMath project targeted a sample size of 10,000 subjects because similar studies of introductory college STEM courses provided enough statistical power to calculate standardized coefficients as low as 0.02 at $p \leq .05$, 0.03 at $p \leq .01$, and 0.04 at $p \leq .001$ (Tai et al., 2006). With a goal of a nationally representative sample of students taking and institutions offering introductory college calculus, the FICSMath project used two stratification criteria to classify the institutions surveyed: type of institution (2 or 4 year) and size of the institution (small, medium, or large). Using the NCES Integrated Postsecondary Education Data System (IPEDS) list of 1,668 two-year and 2,637 four-year U.S. degree-granting institutions and their fall enrollment numbers, cut-offs were established to partition schools into three equally sized populations of students based on enrollment. The cut-offs were similar for 2- and 4-year institutions: fewer than or equal to 5,400; between 5,401 and 14,800; and greater than 14,800 undergraduates. This classification resulted in six lists of institutions stratified by type and size: 2,089 small 4-year colleges, 348 medium 4-year colleges, 200 large 4-year colleges, 1,279 small 2-year colleges, 289 medium 2-year colleges, and 100 large 2-year colleges.

From the IPEDS population of 4,305 institutions, recruitment began with the randomization of each subpopulation list of institutions stratified by size and type. Proceeding down the lists, recruiters used online information to determine if the school offered an introductory calculus course, and if so, they contacted each institution by email or telephone. The preference was to contact the faculty member with overall responsibility for the introductory calculus course or courses directly and, if he or she could not be reached, to contact the mathematics department chair. Using the emails of all those who were teaching calculus, recruiters attempted to engage each one in the project if the faculty coordinator for calculus or the department chair did not want to take responsibility for organizing participation at their institution. In some cases, the school's Institutional Review Board conducted its own human subjects certification. Using calculus enrollment information provided by each institution, the relative distribution of introductory calculus students across each of the six stratification bins was estimated. This effort informed continued recruitment until 276 institutions were contacted, which was deemed sufficient to yield 10,000 calculus students total. Ultimately, 33.2% of the sample were students who attended 2-year schools, and 66.8% were students who attended 4-year schools. Few small 2- and 4-year schools were found to offer calculus (serving, respectively, 2% and 7% of all calculus takers), which is reflected in the sample with 1.8% of the participants at small 2-year schools and 8.3% at small 4-year schools. The bulk of the students taking calculus went to medium and large institutions. This sample, recruited from randomized lists of institutions, was deemed to reflect national enrollment statistics well, with the possible exception that the sample contained a lower percentage of students in medium 4-year schools compared with the percentage of those students in the population (according to our extrapolation): 23.0% versus 39.9%.

Of the 276 institutions with which we established contact, 182 (65.9%) initially agreed to participate. In the end, usable student surveys were received from 134 institutions (i.e., 73.6% of those who agreed to participate or 48.6% of all contacted institutions). The surveys were administered in class within the first few weeks of the 2009 fall term, which was conducive to a very high student participation rate. Professors added each student's grade (or whether they withdrew) to each survey at the end of the semester.

In all, 10,437 students participated (of which 10,082 completed the course and earned a grade), representing a great diversity of educational pathways into introductory college calculus. Because the purpose of this study was to investigate the impact of precollege course decisions among U.S. students, students with the following characteristics were excluded from our analysis. Counts are in parentheses:

- Students who attended high school abroad (788) or were homeschooled (215).
- Students who had previously taken college calculus and were repeating it for a second time, mainly due to earning a low grade or withdrawing from an earlier course (1,619).
- Students with a lapse of 5 or more years between taking college calculus and taking their most advanced high school mathematics course (1,345).
- Students who were not undergraduates (i.e., who were graduate students (78) or high school students (377) taking college calculus for "dual credit").
- Students who did not take precalculus in high school and so were unprepared to directly matriculate into college calculus (1,921).*
- Students whose high school did not offer any type of calculus course (1,002).*

These exclusions served to create a more homogeneous and still large sample of 6,207 students of 216 instructors at 133 institutions. Categories marked with an asterisk were not excluded when tracing the path of students to college calculus but were excluded from predictive models.

Dependent Variable

The dependent variable used in this study was the grade awarded by the professor in introductory college calculus. Certainly, there are other possible outcome measures, among them an administration of a single test of calculus knowledge to all students (Epstein, 2007). Yet, although introductory calculus courses are similar in coverage, differences in emphasis by professor or by the textbook used would make a single exam problematic. Instead, using the actual grade earned has the advantage of real-life validity. It reflects a professor's best judgment of each student's mastery of content and carries with it the implicit encouragement or discouragement to continue in a STEM discipline or enroll in the next level course (Schwartz, Sadler, Sonnert, & Tai, 2009). Institutions participating in the sample also used different measures in awarding grades, with 28% using single letter grades (i.e., A, B, C, D, F), 35% using "+" and "-" to augment letter grades (e.g., A+, B-), and 37% using a 100-point scale. All letter grades reported were converted to the same 0- to 100-point scale (e.g., B+ = 88, B = 84.5, B- = 81). An "A+" grade was coded as 98 in colleges that awarded it; otherwise, an "A" was coded as 95. For uniformity, all grades below 60 were coded as 55, representing an "F." The mean grade was 80.7 (SD = 12.0). Skewness was -0.57, showing a moderate amount of "ceiling effect," with higher grades being awarded more frequently than lower grades.

Independent Variables

Independent variables used in this study include variables that account for differences in students' mathematics background (e.g., the courses that they took in high school, grades earned, and standardized test performance) and those that involve matters having nothing to do with mathematics per se but may explain some of the differences seen in introductory college calculus performance (e.g., career interest, gender, race and ethnicity). Potential variables were identified though a review of the research literature on performance in college mathematics, through consultation with the FICSMath advisory panel of professors and teachers, and through online surveys of both high school mathematics teachers and college mathematics professors. Efforts to model how demographic differences between students relate to performance in STEM courses and selection of majors show the wisdom of accounting for race, gender, college year, and parental-education level as a proxy for socioeconomic status (Sirin, 2005; Tai & Sadler, 2001; Trusty, 2002). Frequencies relating to these demographic variables are reported in Table 1.

Race and gender. Race was accounted for through six categories, with the survey listing five options: White, Black, Asian, Native American or Pacific Islander, and Other. Students who selected more than one category were assigned to a new category, Multiple. With respect to gender, the survey listed the options of female and male, and gender was coded as a dummy variable (0 = female, 1 = male).

Year starting algebra. Another variable, year starting algebra, accounted for the year in which the student took Algebra I. This first mathematics course in the sequence to calculus taken in middle or high school was represented by a variable with possible values of 7, 8, 9, 10, 11, or 12. Although Grade 7 was not an option on the survey, it was presumed that students who took geometry or Algebra II in eighth grade took Algebra I in seventh grade.

Year in college. The year in which calculus is taken in college is an indicator both of readiness for a rigorous STEM course and student maturity. Because differences in STEM major enrollment have been noted based on year in college (Moreno & Muller, 1999), we accounted for this as a binary variable, with calculus being taken in either the first year of college or the second year or later. We explored the use of more detailed categories of year in college (i.e., first, second, third, fourth, and later), but no statistically significant performance difference was found among students taking calculus after the first year of college. Hence, a binary variable was deemed sufficient.

College precalculus. Another variable accounted for was students' enrollment in college precalculus. Many students must take college precalculus, which delays their entry into college calculus by at least a semester and often a year. This assignment is often the result of performing poorly on a college mathematics placement exam (Hsu & Bressoud, 2015). Hence, being assigned to this category is also a measure of being less prepared for calculus. Such an interpretation is supported by the fact that those who had taken college precalculus had a mean score on the SAT/ACT of 570 (ACT actual mean of 25), and those who had gone to college calculus directly had a mean score of 632 (ACT actual mean of 28). A concordance was used to equate the SAT and ACT test scores (Dorans, 1999).

Table 1
Sample Demographics

Variable	Values or categories	Subjects	% of total
Race	Asian	539	8.68%
	Black	254	4.09%
	Multiple	36	0.58%
	Other	383	6.17%
	Native American or Pacific Islander	44	0.71%
	White	4,951	79.76%
Gender	Female	2,253	36.30%
	Male	3,954	63.70%
Parental education	Mean ≤ .5 (no high school degree)	235	3.79%
	.5 < Mean ≤ 1.5 (high school graduate)	1,227	19.77%
	1.5 < Mean ≤ 2.5 (some college or associate's degree)	2,018	32.51%
	2.5 < Mean ≤ 3.5 (4-year degree)	2,153	34.69%
	3.5 < Mean (graduate degree)	574	9.25%
Year starting algebra	7	230	3.71%
	8	3,568	57.48%
	9	2,344	37.76%
	10	53	0.85%
	11	11	0.18%
	12	1	0.02%
Year in college	1	4,132	66.57%
	≥ 2	2,075	33.43%
College precalculus	No	5,014	80.78%
	Yes	1,193	19.22%
Career interest	All other	1,804	29.06%
	Medicine	1,073	17.29%
	STEM	3,330	53.65%
High school calculus	No	3,019	48.64%
	Yes	3,188	51.36%

Note. Demographics are for subjects meeting inclusion criteria, $n = 6{,}207$.

Career interest. The intended career of students was represented by the categories of STEM, medicine, and all other. For most students pursuing a STEM major, calculus is required; many medical schools also require calculus. The incentive for earning a good grade in calculus may be higher for STEM majors and for those intending to go to medical school.

Parental education. Highly educated parents are a proxy for high socioeconomic status and the benefits that it brings: help with homework, positive attitudes toward mathematics, attending well-funded schools, and experiencing extracurricular educational enrichment. Parental education was represented by a continuous variable indicating the average level reached by both parents (on a 0- to 4-point scale: 0 = < high school graduate, 1 = high school graduate, 2 = some college or associate's degree, 3 = 4-year degree, and 4 = graduate study). For students reporting only one parent's education, that single value was used. The mean value of parental education was 2.4 (*SD* = 1.0), between some college and a 4-year degree.

High school calculus. Taking high school calculus was represented by a single binary variable. Students who took calculus were coded as 1 and those who did not as 0. Although there are different levels of calculus (e.g., non-AP, AP AB, AP BC) and different measures of performance in these courses (e.g., AP exam scores, grades awarded by teachers), we did not make such fine-grained distinctions but, instead, used the broad indicator of exposure to a year of calculus in high school.

Preparation for calculus. Variables arising from high school course taking were easily assigned to two categories: preparation for taking calculus and taking high school calculus itself. Preparation for calculus variables were the grades students received in the collection of courses that students generally take prior to calculus (i.e., Algebra I, geometry, Algebra II, precalculus) and their SAT or ACT quantitative score (neither of which includes any calculus items). All of these related variables were incorporated into a composite variable that reflected precalculus preparation. This composite was normalized, with a mean of zero and a standard deviation of 1.0, which allowed for easier interpretation of the final regression models. The details of constructing this composite are presented below. From each student's reporting of high school mathematics coursework year and grade, the sequence of courses was determined and summary statistics calculated across all subjects. The resulting aggregated paths were visually represented in a Sankey diagram (Sankey, 1896).

Missing Data

To prevent students with missing data from being deleted from the sample (the variable with the largest percentage of missing data was SAT and ACT scores, of which 14% were missing), multiple imputation methods were used (Allison, 2002; Rubin 1976, 1987, 1996). Based on the Markov Chain Monte Carlo (MCMC) approach (van Buuren, 2007), we began with initial values of the missing values sampled from a normal distribution similar to that of the existing values for that variable. Regression was used to estimate the dependent variable using all of the other variables as independent variables. After 20 such samplings and regression estimates, a mean was calculated and substituted for the missing value (XLStat, 2016).

Statistical Modeling

Although care was taken in the sampling of colleges to produce a nationally representative data set, differences remained among the many colleges in terms of grading policies and professors' stringency in the awarding of grades. This situation necessitated an approach that would control for institutions and for instructors within the same institution. A dilemma was posed by the hypothetical possibility of students who exhibited identical mastery of calculus concepts and skills on homework and tests but had earned different grades at different institutions or in different professors' classrooms. This is illustrated by the comparison of grades earned by students in college calculus with the mean grade awarded by instructors and awarded within each institution (Figure 1). Although the overall means of the average grades by institution and by instructor were very similar (80.09 [*SD* = 6.34] for 134 institutions and 80.20 [*SD* = 6.63] for 338 instructors), it is likely that professors employed different levels of stringency in grading and that institutions exhibited different grading policies.

Because a basic assumption of regression is that every observation is independent of each other, clustered data are a violation of this assumption. Students in a single professor's classroom are generally more similar than students between professors' classrooms, and professors' classrooms within an institution are more similar than those between institutions. Because of this nested data structure (i.e., student, instructor, institution), we employed HLM with the three levels of

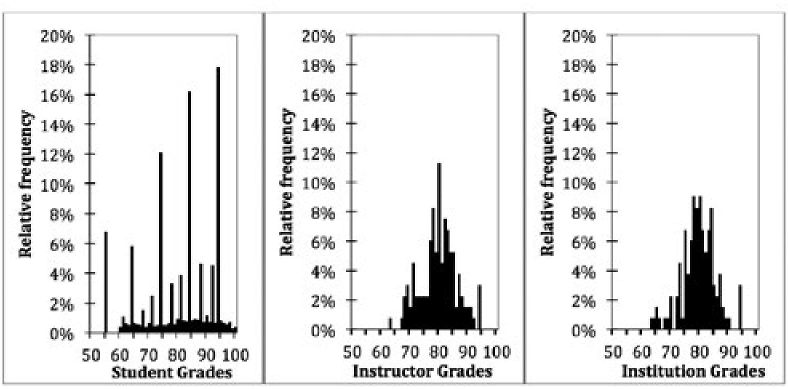

Figure 1. Distribution of grades in college calculus by student, instructor, and institution.

students, instructors, and institutions. This also helped to control for the differing numbers of students in each professor's classroom and in each institution; otherwise, classes with a greater number of students would have had an outsized influence in the regression analysis.

We produced six HLMs, starting with an unrestricted means or "empty" model that explained student grades and accounted for the variability between each professor and for the variability between each institution. This empty model can be thought of as including the mean grade awarded by each professor and the mean grade for each institution. In the case of only one professor participating at an institution (47% of the institutions), this mean is the same. No additional variables are present in this model, thereby allowing the estimation of the amount of variance explained at each hierarchical level. We then built models with increasing numbers of variables, the high school calculus variable, and the calculus preparation composite (and its square), adding each separately to calculate the increment in the multiple coefficient of determination for each and then for all of them together. Finally, we computed a "main effects with interactions" model. We also explored nonlinear aspects of the variables in the models.

To examine goodness of fit of the models, we use a pseudo-R^2 statistic calculated as the squared correlation between the actual and the predicted grades (Singer & Willett, 2003). An increment-to-pseudo-R^2 test calculates the significance of increases to this statistic. Both the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) were also used to examine model fit (McCoach & Black, 2008). The BIC penalizes the number of parameters more strongly in the model than does the AIC. These statistics help to establish whether increasingly sophisticated models (with more variables) provide a better fit to the data.

Results

In this section, we aggregate subjects' histories of math course taking to identify the common pathways leading to college calculus. We show how a calculus preparation composite variable was constructed using grades awarded to subjects in relevant high school courses and their standardized test scores. We then build several regression models using the calculus preparation composite variable to predict college calculus grades, focusing on the impact of high school calculus.

The Path to College Calculus

Using the entire data set, it is feasible to describe the pathways that students take to college calculus (Jaffe, 2012). We calculated the percentages of students enrolled in a particular mathematics course in each year of high school (Table 2). Seventh-grade Algebra I enrollment was estimated from eighth-grade Algebra II and geometry enrollments. Nearly all students who subsequently enrolled in calculus had 4 years of mathematics in Grades 9–12. Most (83%) had taken precalculus, and half had taken calculus in high school.

By calculating the number of students who moved from course to course in high school, we produced a diagram denoting the path that students followed on their way to college calculus (Figure 2). Clearly evident were two "main sequences" beginning with Algebra I: one starting in eighth grade and progressing through geometry, Algebra II, and precalculus to calculus or other mathematics courses and the other starting with Algebra I in ninth grade and progressing to precalculus in 12th grade. Yet, there were many students who did not follow these routes (McFarland, 2006). One in four students reaching high school precalculus skipped either geometry or Algebra II after Algebra I or took a combined course (which has been coded as the higher level in the common sequence). Some form of high school calculus had been taken by 52% of students enrolled in introductory college calculus. In the study sample, 62% of students who started Algebra I in eighth grade and 21% of students who started Algebra I in ninth grade took calculus by the end of high school. Among students who started algebra in ninth grade, most took a later course that combined Algebra II and precalculus or skipped a geometry course entirely.

Of all of the college calculus students surveyed, many took precalculus in college: 5% of all students for the first time, 17% after taking precalculus in high school, and 6% after taking calculus (and precalculus) in high school. The large

Table 2
Course Taking in High School by Grade Level

Course type	Course name (abbreviation)	7	8	9	10	11	12	Total
Preparation for calculus	Algebra I (Alg. I)	6%	44%	38%	3%	1%	0%	92%
	Geometry (Geo)		5%	46%	38%	3%	0%	93%
	Algebra II (Alg. II)		1%	14%	47%	27%	2%	93%
	Precalculus (Pre-Calc)		0%	1%	6%	49%	27%	83%
Calculus	Non-AP (Calc)		0%	0%	0%	2%	19%	21%
	AP AB (Calc AB)		0%	0%	0%	3%	23%	26%
	AP BC (Calc BC)		0%	0%	0%	1%	4%	5%
Other	Integrated Math (Int Math)		1%	1%	3%	6%	3%	14%
	Other math (Other)		0%	0%	0%	2%	6%	8%
Statistics	Non-AP (Stat)		0%	0%	1%	3%	6%	11%
	AP (AP Stat)		0%	0%	0%	2%	6%	9%
Total			51%	99%	100%	99%	96%	

Note. n = 8,933. Totals may not add up due to rounding.

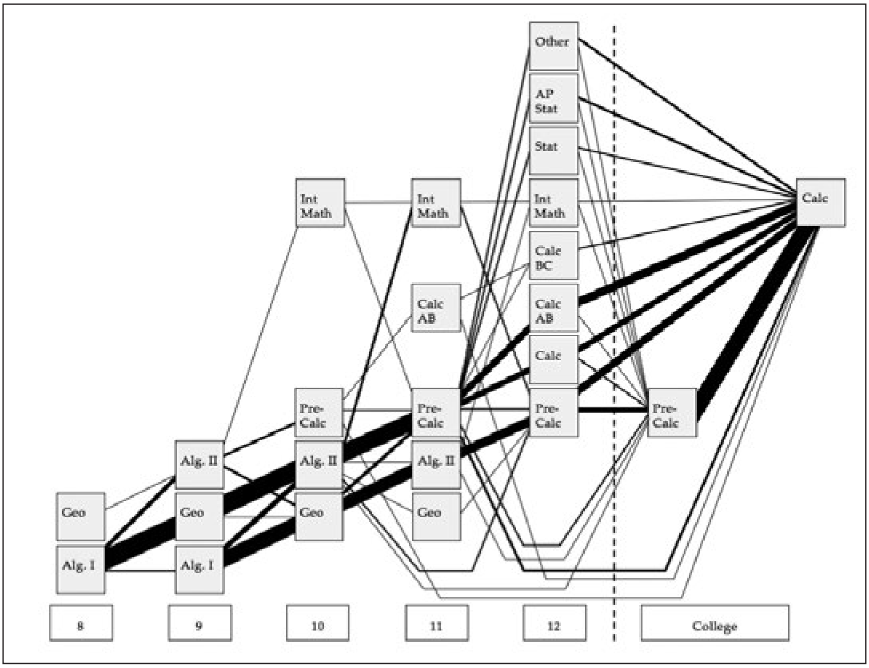

Figure 2. The path to college calculus. Paths of <1% of the total are not displayed. Line widths are proportional to the number of students in each path. Abbreviations used are listed in Table 2.

number of students who retook precalculus in college is surprising. A nationally representative federal study (Rasmussen et al., 2011) found that 31% of all students who had taken a calculus course in high school enrolled in college precalculus, casting doubt on Betebenner's (2001) claim that a high school mathematics course prepares students for the same level course in college. Hsu and Bressoud (2015) reported that retaking precalculus is commonly the result of students' poor performance on a college mathematics placement exam. Many colleges employ standardized tests designed specifically for college placement in mathematics (e.g., Accuplacer®, ACT Compass®). Others use SAT or ACT scores without proper attention paid to establishing cut scores or institution-developed exams given in the summer before matriculation that may be poorly designed (Fitchett et al., 2011). Colleges that use a combination of measures claim better mathematics placement success (Denny, Nelson, & Zhao, 2012; Madison et al., 2015).

Calculus Preparation Composite Variable

To compare the role of high school courses that prepare students for taking calculus (versus high school calculus courses themselves), we first explored the relevant measures of achievement: the grades earned in preparatory high school courses and standardized test scores. Correlations between the six variables relating to preparation for taking calculus are shown in Table 3. Although correlations between each variable and college grade were only weak in magnitude, some moderately strong correlations were exhibited between variables, particularly the high school mathematics grades earned. This pattern warrants concern about the multicollinearity of these variables, if they are all entered in a multiple regression model, and suggests that the construction of a composite variable may be appropriate. Combining variables into a single composite often produces robust and more finely graduated measures that exhibit a higher degree of normality than a single variable. Moreover, using multiple measures may more accurately reflect student preparation.

Figure 3 provides a visual way to understand how each of these six variables behaves alone as a predictor of college calculus performance. The area of each symbol is proportional to the number of students in each group. All of these variables show a high degree of monotonicity; that is, those with higher grades in prior mathematics courses or higher SAT or ACT scores earned higher grades in college calculus, on average. Of special note is that the SAT or ACT test is typically taken late in the spring of junior year or early in the fall of senior year in high school (prior to applying to college), in large part measuring mathematical knowledge prior to that acquired during a student's senior year in high school. A conclusion that can be drawn from Figure 3 is that each of these six variables increases with college grade; hence, any of them could be used alone to predict college calculus performance. Inspection of Figure 3 also reveals that students enrolled in college calculus earned high grades in their high school mathematics courses, with the most common grade of "A" exhibited by the area of each data bubble. With this ceiling effect of earning high grades, combining these measures into a composite provides a smoother, more Gaussian distribution (shown in the histogram in Figure 4).

Table 3
Correlations Between Preparation for Calculus Variables and College Grade in Calculus

Variable	College grade	Algebra I grade	Geometry grade	Algebra II grade	Pre-Calc grade	SAT	ACT
College grade	**1**	.208	.214	.243	.310	.154	.229
Algebra I grade	.208	**1**	.429	.467	.325	.094	.180
Geometry grade	.214	.429	**1**	.466	.386	.099	.206
Algebra II grade	.243	.467	.466	**1**	.494	.092	.195
Pre-Calc grade	.310	.325	.386	.494	**1**	.124	.217
SAT	.154	.094	.099	.092	.124	**1**	.138
ACT	.229	.180	.206	.195	.217	.138	**1**

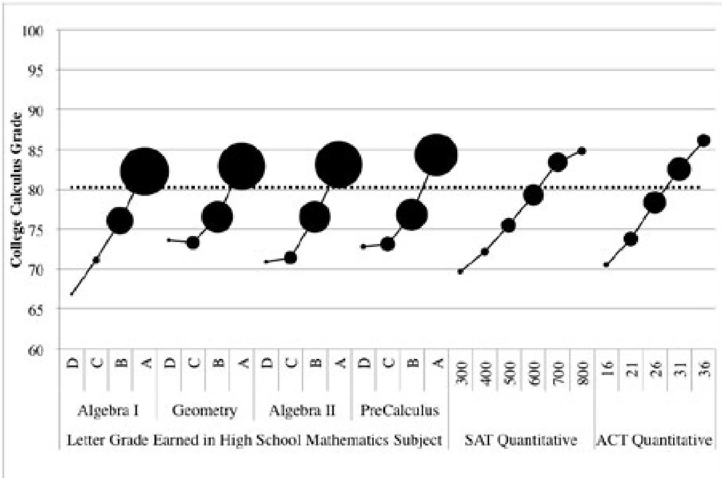

Figure 3. Relationship between grade earned in college calculus and individual variables comprising the calculus preparation composite. The symbol area is proportional to the number of students in each group. The dotted horizontal line represents the mean grade in college calculus.

The relationship between the preparation for calculus variables was further explored by carrying out a multiple linear regression on college calculus grade (Table 4). The effect of any given independent variable represents variance in the dependent variable (i.e., grade in college calculus) that is accounted for after the variance associated with all other independent variables is controlled. The preparation variables were all statistically significant in predicting college calculus grade. Standardized coefficients allow us to compare effect sizes between variables by representing the change in college calculus grade associated with every *SD* change in the variable. Precalculus grade has the largest standardized coefficient, which makes sense because it is the course taken just prior to calculus for most students.

The construction of a calculus preparation composite was additionally supported by performing a factor analysis on the six variables above. A single-factor solution emerged, combining all six variables, in accordance with the Kaiser-Guttman rule (Kaiser, 1991): The first factor had an eigenvalue greater than 1.00 (i.e., 1.924), and subsequent factors showed a clear "elbow," with their eigenvalues being much smaller than 1.00 (0.208, 0.152, 0.044, 0.003). The single-factor solution had loadings for each variable greater than 0.400. The composite was calculated by averaging the standardized components and then standardizing the average again, with a mean of 0 and a standard deviation of 1, for ease of interpretation. The variable was more normal and exhibited less of a ceiling effect than many of the variables from which it was constructed, with a skewness of -0.57 (see histogram in Figure 4). One should keep in mind that the composite is not an unadulterated measure of preparation for calculus. Instead, it may contain other elements

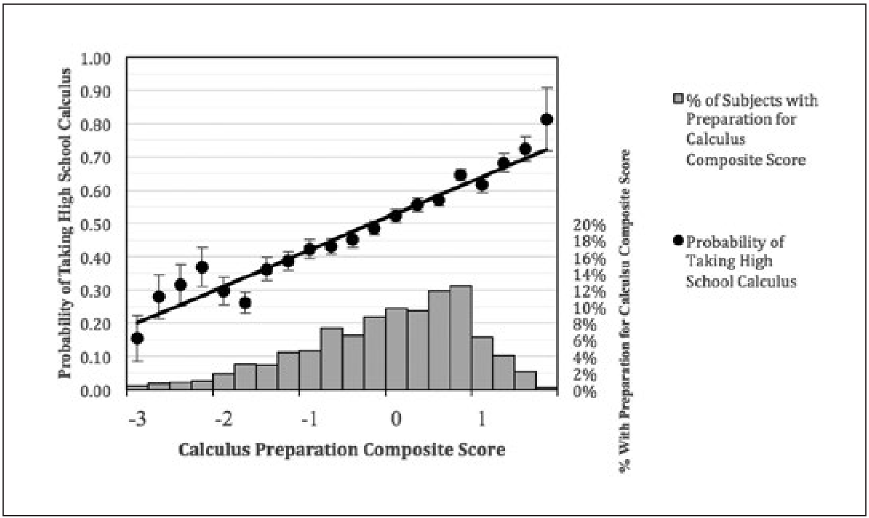

Figure 4. Relationship between calculus preparation composite and probability of taking high school calculus.

(e.g., so-called "grit," good study habits, school attendance, peer support, parental interest in mathematics) with which it may be correlated.

Figure 4 shows the probability of enrolling in high school calculus as a function of students' calculus preparation composite, grouping students into bins for each 0.25 *SD*. Those with strong preparation in coursework prior to being able to take high school calculus had a higher probability of taking calculus in high school. A best-fit linear function models the data well ($R^2 = .941$). The half of students with a low value of the composite (less than zero) took a calculus course in high school with a 41% probability, and those with a high value (greater than zero) took a calculus course in high school with a 60% probability.

Table 4
Regression Model for the Components of the Calculus Preparation Composite

Source	Coefficient	*SE*	Standard coefficient	*SE*	Pr > |*t*|
Intercept	-11.911	3.255			.0003
Algebra I grade	0.153	0.034	0.062	0.014	< .0001
Geometry grade	0.100	0.031	0.045	0.014	.0014
Algebra II grade	0.125	0.033	0.057	0.015	.0001
Pre-Calc grade	0.412	0.029	0.201	0.014	< .0001
SAT	0.015	0.002	0.094	0.012	< .0001
ACT	0.483	0.042	0.141	0.012	< .0001

Employing the variables discussed above, we constructed HLMs using a restricted maximum likelihood estimation. All student-level variables were fixed effects, and institution and instructor were random-effect variables.

Model 0, the unrestricted means model, only included professors nested within institutions. At these two levels, 17.92% of the variance in college calculus grade was located. This result affirmed that grading stringency was important to control for because a considerable amount of the variation in grade was accounted for by differences between institutions and professors.

Model 1 added the background variables of student career interest, gender, race, parental education, year in college, and year starting algebra. F-tests established that all included variables were statistically significant at the $p \leq .05$ level. The significance of institution-level differences was diminished, partly accounted for by students' characteristics.

Model 2 added the variable of interest, high school calculus, increasing the amount of variance explained by 3.8% to 27.3%. F-tests established that all included variables were statistically significant at the $p \leq .05$ level.

Model 3 included all variables in Model 1 and added preparation for calculus as well as its square, increasing the amount of variance explained by 9.1%. Note that the high school calculus variable was not included. By comparing the increment in variance explained from Model 1 to Model 2 and from Model 1 to Model 3, we can see that the inclusion of preparation for calculus variables (i.e., the linear term and the quadratic) explains 2.4 times the variance of high school calculus. F-tests established that all included variables were statistically significant at the $p \leq .05$ level. The calculus preparation composite (and its square) and taking high school calculus were all statistically significant. Because these two variables were somewhat correlated (0.28), as also seen in Figure 4, tolerance statistics for each were calculated to assess collinearity. The variance inflation factors (VIF; 1.28 and 1.39) were well below the minimum level of concern (2.0; Kutner, Nachtsheim, & Neter, 2004).[2]

Model 4 included all variables and shows a 2.7% increase in variance from Model 3. Hence, the inclusion of high school calculus in Model 3 increases the variance explained by less than the 3.8% seen in Model 2. This is because the high school calculus variable and the calculus preparation composite (and its square) are somewhat correlated.

Model 5 adds the three statistically significant interactions between the variables. These were the interactions between the calculus preparation composite and the high school calculus dummy variable, between the calculus preparation composite and

[2] Nonetheless, the weak collinearity between the two independent variables of interest (taking high school calculus and preparation for calculus) might compromise the precision of the parameter estimates. Using propensity weighting, we equalized the groups of high school calculus takers and high school calculus nontakers on all independent variables so that no collinearities existed between taking high school calculus and preparation for calculus (or any of the other independent variables). It turned out that the naturally occurring collinearities did not pose a major threat. The results of the propensity-weighted models were very similar to the results of the unweighted models reported in the main body of the article.

the year in college, and between the high school calculus dummy variable and the year in college. An increment-to-pseudo-R^2 test indicated that the interaction model was significantly superior to the main effects model ($F_{3,5713} = 9.47$, $p < .0001$). Figure 5 presents a visual representation of the interactions in Model 5 and shows the differing degree of benefit of a high school calculus course as a function of the calculus preparation composite for students taking calculus in their first year of college and for those taking it later. For students at all levels of the calculus preparation composite, those who took a high school calculus course earned a higher grade in college calculus, on average. Those at the lower end of the preparation scale appeared to benefit the most from high school calculus, whereas those at the higher end of the scale appeared to benefit to a lesser degree.[3]

Those students who were enrolled in college precalculus did not, on average, appear to derive a benefit from this experience; there was even a negative coefficient of 0.98 points. This result is similar to that of Sonnert and Sadler (2014), who found that a prior semester of college precalculus did not help students' grades when later taking college calculus. Female students earned a calculus grade that was 1.33 points higher than male students, on average. The parental education variable added 0.64 points for each level of parental education beyond "did not finish high school." Compared with first-year students, calculus grades earned by those in their second year or beyond were 1.11 points lower. There was a difference in grade by career interest, with students pursuing STEM or medicine averaging 1.23 or 1.51 points higher than others, respectively.

Discussion

Mathematics professors are correct in that the students in their introductory college calculus classes vary considerably in their mastery of the high school mathematics considered prerequisite for the study of calculus and that students who achieve greater mastery of precalculus concepts and skills perform considerably better when they reach college calculus. It is also important to note again that the selection of variables for inclusion into the calculus preparation composite (Table 3) was supported by a regression model as well as a factor analysis. This means that the performance in earlier precollege courses persisted as a predictor, even when accounting for later courses. For instance, how a student did in Algebra I was still a statistically significant predictor 5 or 6 years later in college calculus. A plausible alternative to this finding would have been that earlier performance

[3] A further hypothesis was explored at the suggestion of peer reviewers of this article. This hypothesis concerns the graduation policy of the state in which students graduated high school (Reys, Dingman, Nevels, & Teuscher, 2007). It was thought that students who enter high school having already taken Algebra I would have to take calculus or another advanced course if they were required to take 4 years of high school mathematics. Students in states without this requirement might take high school calculus based more on interest than on a requirement. The state requirement of the number of years of mathematics necessary for graduation was tested in both main effects and interaction models. It was not found to be significant at the $p \leq .05$ level and, therefore, was not included in the models in Table 5.

Table 5
HLMs for College Calculus Grades

Variable	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5
HLM variance						
Institution	5.8% (1.8)***	3.6% (1.6)*	3.1% (1.6)*	3.2% (1.7)*	3.1% (1.7)*	3.1% (1.7)*
Instructor	9.3% (1.5)***	11.8% (1.7)***	13.4% (1.9)***	14.2% (1.9)***	15.7% (2.1)***	15.7% (2.1)***
Subject	84.9% (1.6)***	84.6% (1.6)***	83.4% (1.5)***	83.6% (1.5)***	81.2% (1.5)***	81.1% (1.5)***
Intercept	79.84 (0.40)***	98.66 (2.21)***	84.15 (2.31)***	94.27 (2.09)***	82.49 (2.19)***	83.12 (2.64)***
Career interest						
Other (baseline)						
Medicine		1.76 (0.44)***	1.79 (0.43)***	1.47 (0.42)***	1.52 (0.41)***	1.49 (0.41)***
STEM		1.37 (0.36)***	1.30 (0.35)***	1.28 (0.34)***	1.23 (0.33)***	1.18 (0.33)***
Race						
White (baseline)						
All other		-2.54 (0.56)***	-2.36 (0.54)***	-1.72 (0.56)**	-1.62 (0.52)**	-1.65 (0.52)**
Asian		0.22 (0.55)	0.12 (0.53)	0.11 (0.51)	0.03 (0.50)	0.03 (0.50)
Black		-3.76 (0.72)***	-3.82 (0.70)***	-1.90 (0.68)**	-2.07 (0.67)**	-2.20 (0.67)**
Calculus in first college year		2.54 (0.38)***	1.88 (0.37)***	1.57 (0.36)***	1.09 (0.35)**	0.33 (0.44)
Parental education		0.62 (0.15)***	0.63 (0.15)***	0.63 (0.14)***	0.63 (0.14)***	0.63 (0.14)***
Gender		-2.24 (0.31)***	-2.21 (0.30)***	-1.30 (0.29)***	-1.33 (0.28)***	-1.30 (0.28)***
Year starting algebra		-2.41 (0.25)***	-0.94 (0.26)***	-1.95 (0.23)***	-0.76 (0.24)**	-0.77 (0.24)**
College precalculus		-2.34 (0.44)***	-1.61 (0.44)***	-1.54 (0.42)***	-0.98 (0.41)*	-1.21 (0.41)**
High school calculus			5.41 (0.31)***		4.50 (0.30)***	3.27 (0.50)***

Variable	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5
Preparation for calculus				4.44 (0.16)***	4.17 (0.16)***	3.75 (0.26)***
(Preparation for calculus)^2				0.66 (0.09)***	0.62 (0.09)***	0.55 (0.10)***
Interactions						
Calculus in first college year* Preparation for calculus						1.16 (0.29)***
High school calculus* Preparation for calculus						-0.71 (0.28)*
Calculus in first college year* High school calculus						1.83 (0.59)**
Goodness of Fit measures						
r^2	17.9%	23.5%	27.3%	32.6%	35.3%	35.6%
Δr^2 from Model 1			3.8%	9.1%	14.8%	15.1%
AIC (smaller is better)	47910.1	47483.9	47221.6	46763.8	46541.0	46516.6
AICC (smaller is better)	47918.1	47511.9	47221.6	46763.9	46541.1	46516.8
BIC (smaller is better)	47918.1	47512.0	47265.0	46810.2	46590.3	46574.6

$^*p \leq .05$. $^{**}p \leq .01$. $^{***}p \leq .001$.

was subsumed in the grades of later high school courses, which was not the case. Teachers of early mathematics courses in high school (i.e., Algebra I and geometry) should be aware that they may have a substantive impact on later performance when students take college mathematics. How well students learn the elements of algebra and geometry in these more basic high school mathematics courses plays out later because some of these fundamental skills and concepts are drawn upon in learning calculus. However, one should also recognize that grade in high school precalculus had the largest standardized coefficient for all variables (Table 4), higher than SAT and ACT mathematics scores, which are often used for college mathematics placement decisions (and college admission decisions).

Figure 5 shows a clear benefit, on average, of taking high school calculus for students, whether they take calculus in their first or later years of college. For college calculus students with a strong preparation for high school calculus, the benefit that they receive from high school calculus may be due to their understanding of more advanced topics (e.g., trigonometry and other functions) and, to a larger degree than for less prepared students, due to learning skills and concepts of calculus itself. Perhaps students with different degrees of preparation gain different skills and concepts from a high school calculus course. Those with a weak background in areas considered preparatory (e.g., algebraic manipulations, graphing functions) may have many of these weaknesses ameliorated, which would show up as a benefit when they later take college calculus. Taking high school calculus is equivalent to a boost of students' college calculus grades of half a letter grade, on average (Figure 5). Students with a relatively weak background in the mathematics considered preparatory for high school calculus appear to benefit even more from participating in high school calculus than do their more well-prepared classmates. We have calculated in a separate logistic regression that, on average, students at a given level of the calculus preparation composite are 10.5% less likely to be placed into college precalculus if they have taken calculus in high school. We take from this that high school students should not be prevented or dissuaded from taking calculus in high school on account of a weak performance in earlier coursework. Although they might not earn a high grade in high school calculus, taking the course appears to bolster their performance later in college calculus and decrease their chances of needing to take precalculus in college.

Our analysis found that students followed several different paths on their way to college calculus. A generation ago, options for high school students on their way to college were few, other than when to start taking Algebra I and whether or not to take calculus in their senior year (if a student started algebra in eighth grade). Current options include starting Algebra I as early as seventh grade; taking Algebra II before geometry; eliminating geometry altogether; opting for Integrated Mathematics for 1, 2, or 3 years; and enrolling in a variety of senior year courses including regular, AP AB, or AP BC calculus, regular or AP statistics, or other mathematics courses. Whereas at one time college mathematics professors teaching calculus could have a good idea of their students' backgrounds because of the few pathways available, the mathematical history of their students is currently more diverse, reflecting a larger number of options in high school courses and their sequence.

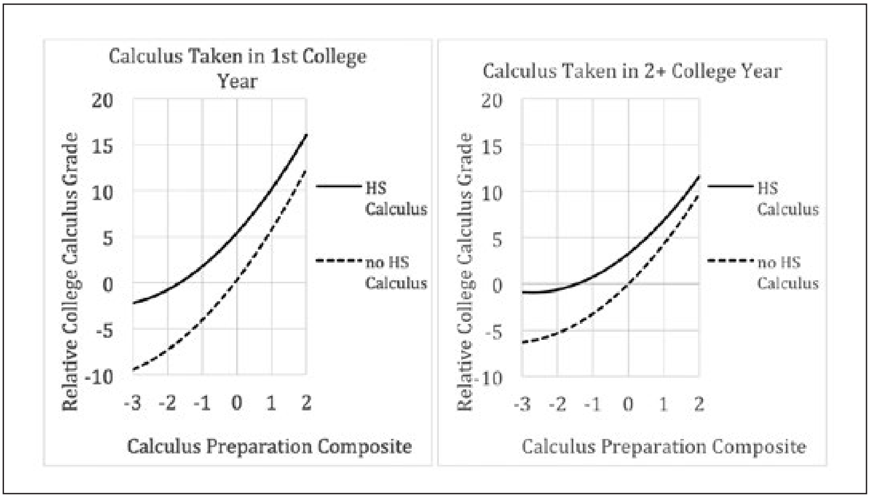

Figure 5. Relationship between college calculus performance, high school preparation, taking high school calculus, and year taking calculus in college.

The concerns of college faculty that high schools were offering calculus courses that were lacking in rigor or "watered down . . . (Leitzel et al., 1987, p. 2)" (Ferrini-Mundy & Gaudard, 1992, p. 57) have largely been countered by the expanded offering of AP calculus courses. Yet, the other clear warning of the time was that affording students the opportunity to take calculus in high school did not leave enough time to "develop a sound background in algebra, functions, and other traditional precalculus topics" (Ferrini-Mundy & Gaudard, 1992, p. 57). A current worry is that this may occur with students who skip geometry in high school or take alternative courses, such as Integrated Mathematics, deemed a "rush to calculus" (Bressoud, 2015, p. 180). We found that of the college calculus students in this study whose highest mathematics course taken in high school was precalculus, 36% had to retake precalculus in college. This was the case even though the mean grade in high school precalculus of this group of "retakers" was a B+. The large number of students required to take college precalculus after also taking it in high school is evidence that many students who imagine themselves well prepared for college calculus are not and that college placement exams are doing a poor job of gauging calculus readiness. Nonetheless, as Bressoud (2015) reported, 80% of college calculus students optimistically stated that their high school mathematics courses provided them with "the knowledge and abilities to succeed in this course" (p. 181) at the start of the term; however, this assessment fell to 55% by the end of the course.

Moreover, the opinion of college faculty that some high school calculus courses may be lacking in rigor may be supported by our finding that many students who had enrolled in high school calculus ended up retaking college precalculus before going on to enroll in college calculus: 23% of those in our sample who took non-AP calculus (who averaged a grade of B+) and 9% of those who enrolled in an AP calculus course (who also averaged a grade of B+ and half of whom reported a score of 3 or higher on the AP exam). This result dovetails with the finding by NCES that 31% of students who had studied calculus in high school enrolled in precalculus upon entering college (Bressoud, 2010b; Rasmussen et al., 2011). Again, this could be the result of college placement tests not adequately assessing student readiness for studying calculus, or it could be a sign of weakness in high school preparation.

Concerning the theoretical perspectives discussed earlier, we find little support for the hierarchical learning theory of Gagné (1968) that was later elaborated on by Jones and Russell (1979). Students who have not mastered the mathematics considered preparatory for calculus are not prevented from learning a great deal when taking high school calculus. Instead, this study provides support for Bruner's (1960, 1971) spiral approach to learning in that being exposed to calculus in high school clearly pays off when students enroll in college calculus. This evidence also supports the view that many college professors' uneasiness about the teaching of calculus in high school may have more to do with a response to a perceived encroachment on their "turf" than with an actual ineffectiveness of high school calculus. Through their highly specialized teaching skills, mathematics knowledge, and credentials, members of these two groups of educators earn differing amounts of professional autonomy, prestige, and compensation (Collins, 1979), with professors obtaining higher occupational prestige and larger average salaries. Turf wars are not uncommon between adjacent professions, such as accountants and tax lawyers (Black & Black, 2004; Munneke, 1999) and radiologists and cardiologists (Dowe, 2006; Drucker & Brennan, 1994; Levin & Rao, 2004). Because many professions require high-level skills with a central cognitive component—in our case, knowledge and teaching of mathematics—these skills become central for the definition of turf. According to Abbott (1988), "many occupations fight for turf, but only professions expand their cognitive dominion by using abstract knowledge to annex new areas, to define them as their own proper work" (p. 102).

In addition to this general tendency of how professions view themselves and others, sometimes guided by selective perception and stereotypes of the "out-group," two specific factors support the observed difference of opinion between high school and college mathematics instructors about the value of high school calculus.

The first factor has to do with differences in the composition of the groups of students they teach. From the high school mathematics teachers' perspective, high school calculus classes are typically taken by the mathematically strongest and most interested students in their schools. Many students are motivated to take such a challenging mathematics class in order to increase their chances of getting accepted into an elite college or university or of earning college credit while in high school (Sadler & Tai, 2007). Teaching calculus—at the pinnacle of the high school sequence—confers prestige and honor to those who are often the most proficient and knowledgeable high school mathematics teachers. As a result of their instruction, many of their best students may be in a position to "place out" of introductory college calculus, particularly those who do extremely well on AP exams. Instructors of college calculus may not have the opportunity to teach these students in their introductory courses and may well underestimate the quality and rigor of high school calculus instruction. Supporting this supposition, national statistics show that of the 291,938 students taking an AP calculus exam in 2008 (either AB or BC), 138,442, or 47%, earned a score of 4 or 5 (College Board, 2008). Yet, students earning a 4 or 5 make up only 29% of those in our data set who reported taking an AP exam. Thus, first-semester introductory calculus instructors in college may be “missing” many of these very high-performing students and have disproportionately more students who have taken calculus in high school but who did not do well enough to “place out” of introductory calculus. These students may exhibit weakness in their knowledge of mathematics considered preparatory for calculus, which may have prevented them from earning high scores on an AP exam.

The second factor supporting the observed difference of opinion between high school and college mathematics instructors about the value of high school calculus is that memory can often be skewed toward the more “extreme” students. These might be students who took high school calculus but are failing spectacularly in college calculus or students who did not take calculus in high school but who are easily mastering the content in their college calculus course because their preparation in high school algebra and precalculus was so solid. These “counterexamples” from the extreme ends of the preparation for calculus distribution stand out and are the students mathematics professors might tend to remember because they are anomalies. A contributing issue here is that, as we have seen, the benefit of high school calculus diminishes at the higher end of the distribution (Figure 5).

This study found that the two variables of interest, mastery of mathematics preparatory to calculus and taking high school calculus, are both powerful predictors of success in introductory college calculus. In essence, both professors and high school teachers are correct. Students who master preparatory coursework do well in college calculus; those with poor preparation do poorly. Those who take calculus in high school also generally benefit; those with weaker preparation benefit even more.

From the findings of this study, we can generate useful observations that are relevant to stakeholders and that might help them contextualize their own experiences:

- College calculus professors: Students taking an introductory calculus course vary widely in preparation, particularly in algebra, geometry, and precalculus, subjects considered essential to learning calculus. This variation explains more about student performance in a calculus course than whether or not they took calculus in high school. However, those who have taken high school calculus—about half—will average a half grade higher in their college calculus courses than students with a similar preparation but without high school calculus. Those who have taken calculus in high school have generally gained a deeper understanding of algebra, geometry, and precalculus. The students who most excelled in their high school mathematics may be missing entirely, having “placed out” of first-semester calculus.

- High school teachers: High school calculus is a course that builds upon knowledge gained earlier in algebra, geometry, and precalculus courses. Yet, even students with relatively weak mathematics backgrounds will generally benefit greatly from taking calculus in high school. Students might later be surprised that they must repeat their highest level high school mathematics course when they get to college even though they earned a high grade in it. Mathematics placement tests often reveal weaknesses that professors want remedied before students are allowed to take college calculus. Even students earning high AP calculus exam grades (i.e., ≥ 3) who intend to major in STEM fields will often retake introductory college calculus.
- Those who are interested in mathematics education policy: Good grades in high school mathematics bode well for a student's experience in college calculus, but they do not guarantee that a student will find college calculus easy. High school calculus is a valuable course for students who are interested in a college major that will require college calculus. It generally gives students a greater mastery of the fundamentals of algebra, geometry, and precalculus. These concepts and skills will be assumed by college calculus instructors to be fully mastered upon entry into their course. High school calculus has the effect of bettering college calculus performance and lowering the chances of being placed into college precalculus (putting students a semester behind in college mathematics). However, there are many ways to bolster the knowledge of mathematics needed for college calculus (e.g., self-study, taking an online course, participation in a summer "bridge" program).

A limitation of this study is that it is not experimental. Indeed, it is hard to fathom how such an experiment would be conducted because it would involve the random assignment of students with varying levels of background to a high school calculus course. Although we have used a well-controlled regression modeling approach, it still cannot prove causality. Other factors, not observed in this study, may be in play. The variables included in the model explain about one third of the variation seen in college calculus grades. Combined, preparation for calculus and high school calculus account for about one sixth of the variance in college grades. Students' study habits, effort expended, anxiety about learning mathematics, membership in a study group, connection with the professor, and use of available tutoring resources, along with nonacademic variables, may contribute to the two thirds of the variance left unexplained by the study.

Another potential limitation of this study is that the sample is limited to those taking college calculus. Those opting out of college calculus, either because of disinterest or taking advantage of high scores on an AP exam, are lost to the sample. Those students who do not at least reach precalculus in high school are also not included in this study. Nearly all such students may enter college needing to take college algebra or precalculus.

Possible future research using the FICSMath data set could include modeling the impact of the different kinds of calculus courses offered by high schools (i.e., non-AP, AP AB, AP BC, dual credit, International Baccalaureate [IB]) and the performance in such courses (i.e., grade earned, AP or IB exam scores) on introductory college calculus performance. Such an analysis could contribute to establishing the performance level in high school calculus that should earn students college credit for calculus. In addition, students differ considerably in the high school courses they take or the order in which they take them. All these variations could be explored to determine if any make a statistically significant difference in college calculus performance.

Conclusion

College calculus students are a diverse group, many having started their high school mathematics sequence in eighth grade (with Algebra I) and half having taken calculus by 12th grade. This study explored the impact on college calculus performance of high school mathematics course taking and performance while controlling for student background in a large, nationally representative sample of students taking college calculus.

Surveys of high school mathematics teachers and college mathematics professors show a substantial discrepancy between their views of how well students are prepared for college calculus. High school teachers generally feel that students are well prepared, especially if they take calculus in high school. Many college professors disagree, arguing for greater time and effort to be spent on the mastery of precalculus and algebra in high school, because a large fraction of students perform poorly on placement tests and must repeat precalculus in college. They hold that taking high school calculus without a mastery of the prerequisites of algebra, geometry, and precalculus is of little benefit. From the theoretical perspective of turf conflict between professions, such a discrepancy of views is to be expected because calculus exists on both sides of the high school–college divide. The professorial profession, whose original turf—calculus—is being increasingly "encroached upon" by high school teachers would appear more likely to view this development with skepticism and lack of social trust (Fisler & Firestone, 2006) than would the teachers who would be more enthusiastic about teaching on this newly gained turf.

However, there is no simple answer to whether access to high school calculus should be withheld in favor of more preparation in lower mathematics or should be championed and expanded. This study found that both calculus in high school and a solid mastery of prerequisite mathematics predict later success in introductory college calculus. Either approach helps those planning to take calculus in college. Mastery of algebra, geometry, and precalculus, on average, has more than twice the impact of a high school course in calculus. Yet, taking high school calculus does not appear to handicap students at all but, rather, provides students with an opportunity to strengthen prerequisite knowledge and skills while introducing some of the content of college calculus. We found evidence that the mathematically weaker students who take high school calculus gain more from it, in terms of their college calculus grades, than do their classmates with stronger preparatory backgrounds.

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Submitted January 17, 2017
Accepted April 3, 2017