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First mathematics course in college and graduating in engineering: Dispelling the myth that beginning in higher-level mathematics courses is always a good thing

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Abstract

Background: Graduation rates in engineering programs continue to be a concern in higher education. Prior research has documented an association between students' experiences in first-year mathematics courses and graduation rates, but the influences of the mathematics courses completed and the grades earned are not fully understood.

Purpose: The purpose of this study was to investigate the relationship among the first undergraduate mathematics course a student completes, the grade they earn in this course, and the likelihood of graduating with a degree in engineering within six years.

Method: The study involved 1504 students from five consecutive cohorts of first-year students enrolled in an engineering degree program at a medium-sized Midwestern public university. Logistic regression was used to model the interrelationship between course and grade in predicting the relative likelihood of graduation for students enrolled in 16 different mathematics courses.

Results: Overall, students who take Calculus I or a more advanced mathematics course as their first mathematic course and who are more successful in their first mathematics course are more likely to graduate with a degree in engineering. However, considering grade and course together, some groups of students who are more successful in lower-level mathematics courses are as likely to graduate as students who are less successful in upper-level mathematics courses.

Conclusions: Evidence from this study helps to dispel the myth that beginning with higher-level mathematics courses is the optimal course-taking strategy when pursuing an engineering degree. Findings have implications for student advising, curriculum and instruction, high school course-taking, and broadening participation in engineering.

KEYWORDS

Calculus, engineering graduation rates, first-year experience, mathematics, regression

1 | INTRODUCTION

The graduation rates of students in engineering programs continue to be a major concern in higher education. Typically, only approximately 50% of students who join an engineering program successfully complete an engineering degree (American Society for Engineering Education [ASEE], 2017; Geisinger & Raman, 2013; Krause et al., 2015; Levin & Wyckoff, 1995; Van Dyken et al., 2015; Yoon et al., 2014). This concern has led many researchers to investigate the factors that influence students' persistence in engineering programs and that sway them to change their majors. Although many factors have been documented to be potentially associated with students' graduation with a degree in engineering (e.g., Geisinger & Raman, 2013), factors associated with students' mathematics experiences continue to surface as important predictors of graduation. In particular, the first mathematics course a student takes as part of their engineering program (e.g., Bowen et al., 2019) and the grade earned in that course (e.g., Krause et al., 2015) have been documented as significant predictors of graduating with a degree in engineering.

Here, we report on a large-scale study of one Midwestern university's graduation rates in engineering and their relationship with students' first mathematics course and the grade they received in that course. This study builds on previous work investigating mathematics courses and grades, and it contributes to the call for more replication studies in engineering education "to build a knowledge base that will increase the relevance and credibility of engineering education research" (Benson & Borrego, 2015, p. 390). Moreover, our study extends prior research by specifically considering the relative likelihood of graduating with a degree in engineering associated with 16 different mathematics courses and the grade received in the courses. In this way, in addition to documenting the success of students who begin with a first course in the Calculus sequence or higher, we were also able to consider the relative success of students in lower-level mathematics courses and the potential for them to succeed even if they do not begin with a course in the traditional Calculus sequence. Findings from this investigation have possible implications for advising, curricular planning, selecting high school courses, and broadening participation in engineering. Moreover, findings from this investigation potentially offer evidence to dispel the myth that starting an engineering program in a higher-level mathematics course is always the best path to an engineering degree.

2 | BACKGROUND FOR THE STUDY

General factors that have been found to relate to student success in engineering programs include both cognitive and noncognitive factors (Jones et al., 2010). In a review of the literature, Geisinger and Raman (2013) identified five groups of factors that influence retention rates in engineering programs: (1) classroom and academic climate, (2) grades and conceptual understanding, (3) self-efficacy and self-confidence, (4) interest and career goals, and (5) race and gender (see p. 917). As an example, in relation to classroom climate, Geisinger and Raman (2013) found that students claimed that the "individualistic culture" (p. 918) of engineering programs focused too much on competition, and therefore some students changed to "more interesting" or "more appealing" (p. 919) programs. In several studies, female students reported the lack of female role models as well as feeling "undervalued" (Geisinger & Raman, 2013, p. 920) by male counterparts and, thus, felt discouraged from continuing in the engineering field (also see Ellis et al., 2016). In addition, other studies have demonstrated the important role of engineering identification (i.e., the extent to which students value engineering) as a significant predictor of students' intentions to remain in an engineering major and pursue an engineering career (Jones et al., 2014, 2016). Also, multiple studies have shown that indicators of achievement such as students' high school grade point average (GPA), SAT scores, or ACT scores are significant predictors of retention (Honken & Ralston, 2013; Levin & Wyckoff, 1988, 1995; Moller-Wong & Eide, 1997; Pembridge & Verleger, 2013; Tyson, 2011). Overall, many factors influence students' decisions to persist in engineering. Following is a discussion of some of the factors found in the literature related to the study of mathematics, culminating in a discussion of the influence of first-year mathematics courses and grades in those courses.

The instructional practices used in mathematics courses can influence student retention and degree completion, particularly whether courses involve student-centered practices or active learning strategies (Klingbeil & Bourne, 2015; Krause et al., 2015; Rasmussen et al., 2019). In their study of first-year mathematics courses in science, technology, engineering, and mathematics (STEM) fields, Rasmussen et al. (2019) found that the most successful Calculus 1 courses "as determined by passing rates; persistence on to Calculus 2; and changes in confidence, interest, and enjoyment of mathematics" p. 100 were the ones in which "instructors…employed a variety of instructional practices that actively engaged students during class" (p. 102). Similarly, Krause et al. (2015) attributed mathematics instructors' "teachercentered, information-transmission attitude and practices" (p. 17) to the negative first-year mathematics experiences related to persistence to graduate. In addition, the President's Council of Advisors on Science and Technology (2012) found that although many students performed well in their introductory STEM courses, they left the STEM field because of the teaching methods and atmosphere, describing them "as ineffective and uninspiring" (p. 5). These studies provide evidence that the quality and method of instruction in mathematics and other introductory STEM courses can influence student retention.

Throughout the literature, variables associated with mathematics achievement appear as recurring correlates of student retention and graduation in engineering. Students' mathematics scores on the SAT or ACT have been found to be predictors of retention or degree completion (e.g., Besterfield-Sacre et al., 1997; Moller-Wong & Eide, 1997; Veenstra et al., 2009; Zhang et al., 2004). For example, Zhang et al. (2004) found that when students' SAT mathematics scores increased by 10 points, they were 3%–8% more likely to graduate with a degree in engineering. Further, mathematics achievement as measured by student grades in mathematics courses has also been found to predict retention and graduation (Budny et al., 1998; Gardner et al., 2007; Krause et al., 2015; Levin & Wyckoff, 1995). For example, by analyzing grades in first-year mathematics courses, Budny et al. (1998) found that higher grades increased the likelihood of retention in engineering irrespective of the mathematics course taken. These studies suggest that students' mathematics achievement serves as an important predictor of success in engineering.

Calculus is a required mathematics course for most STEM programs, including engineering (Bressoud et al., 2015; Rasmussen et al., 2019). It typically serves as a gatekeeper to more advanced courses, resulting in many students leaving engineering programs after their first-year (Bressoud et al., 2015; Geisinger & Raman, 2013; Pembridge & Verleger, 2013; Rasmussen et al., 2019; Van Dyken & Benson, 2019). However, students who begin their study of mathematics in Calculus or higher-level mathematics courses are more likely to complete a degree in engineering than those who begin in lower-level mathematics courses (e.g., Bowen et al., 2019; Krause et al., 2015; Van Dyken et al., 2015). Krause et al. (2015) found that, while controlling for course grade, the odds of a student remaining in an engineering program whose first-year mathematics course was below Calculus I were 0.40 times the odds of students who took Calculus I. They also found that for a student who took a course above Calculus I, the odds of persisting to graduation were 2.3 times the odds of a student who took a Calculus I course. Similarly, Bowen et al. (2019) found that students who were eligible to take Calculus as their first mathematics course were more likely to graduate with a degree in engineering than students who were not eligible to take Calculus as their first mathematics course. Van Dyken et al. (2015) also found that approximately 50% of engineering students who begin in Precalculus and approximately 40% of those who begin in a two-semester Precalculus/Calculus I combination course leave their engineering program after their first-year. In a logistic regression model including grade, course, gender, and race as predictor variables, they found that grade and course level were both statistically significant predictors of retention after one year. Being able to enroll in a Calculus course in the first-year increases the likelihood of graduating with a degree in engineering.

However, upon entering college, many students are not ready to enroll in a Calculus course as their first mathematics course. Mathematics readiness plays a major role in student retention and degree completion in engineering programs (Faulkner et al., 2019; Gardner et al., 2007; Geisinger & Raman, 2013; Greefrath et al., 2017; Madison et al., 2015; Redmond-Sanogo et al., 2016; Tyson, 2011). In general, when students are adequately prepared in mathematics in high school, they are more apt to enroll in advanced mathematics courses in college and might earn higher grades in those courses. Redmond-Sanogo et al. (2016) found that students who did well in high school mathematics (e.g., Precalculus, Trigonometry, and Calculus) were more successful in college Calculus. Sadler and Sonnert (2018) found that taking Calculus in high school predicts later success in college Calculus; however, mastery of mathematics content prerequisite to Calculus (e.g., Algebra and geometry) was found to have more than twice the impact of a high school Calculus course on later success in college Calculus. Yoon et al. (2014) found that students who entered college with transfer credits for Calculus, such as advanced placement (AP) or College-Level Examination Program (CLEP) credits, had higher graduation rates than students who took Calculus at the university level. In general, high school opportunities and preparation in mathematics play an important role in a student's placement and success in their first mathematics course in college.

Across the United States, students' high school mathematics course opportunities and experiences vary greatly. Students' access to advanced mathematics courses is often related to the demographics of the schools they attend (Mendes & Duncombe, 2020; Norwicki, 2016; Oakes et al., 2003). Differences in opportunities to study mathematics perpetuates a racialized social system that supports increased earning potential for Whites when compared with Blacks, Hispanics, and Native Americans (Battey, 2013). Compared to other schools, those that are considered high poverty (i.e., schools with more than 75% Black or Hispanic students) offered fewer advanced mathematics courses (in particular Calculus) or Algebra in 7th or 8th grade, which serves as the gatekeeper to the study of advanced mathematics

(Norwicki, 2016). Similarly, the availability of AP courses is related to the demographics of schools (Oakes et al., 2003) in that the percentage of schools offering AP courses has been shown to be lower for high-poverty schools and schools with at least 75% Black or Hispanic students (Norwicki, 2016). Many capable students do not have the opportunity to study advanced mathematics in high school simply because of the middle or high school they attend, a situation which greatly affects later opportunities for their first mathematics course in college.

The first mathematics course that students enroll in as part of their engineering program has been documented as a significant predictor of student retention and degree completion. In addition, some researchers argue that students' success, as determined by the grade in the first-year mathematics courses, is as important, if not more important, than the course in predicting retention and degree completion (Gardner et al., 2007; Krause et al., 2015; Levin & Wyckoff, 1995; Tyson, 2011; Van Dyken et al., 2015; Veenstra et al., 2009; Yoon et al., 2014). For example, Van Dyken et al. (2015) found that 88.48% of students who received an A in their initial mathematics course were retained one year after starting the program; 86.27% of students who received a B, 78.93% of students who received a C, 74.23% of students who received a D, and only 56.74% of students who received a F were retained. Gardner et al. (2007) found that when considering both the course and grade for students' first mathematics course and controlling for several other background variables, the grade in the course was the only statistically significant predictor of retention as defined by enrollment in an engineering program one year later. They discovered that 60% of students who received an A in their first mathematics course were retained and 73.6% of students who received a B were retained. These rates dropped as grades continued to lower: 67.5% of students with a C, 48.9% of students with a D, and only 29.2% of students with an F were retained. Yoon et al. (2014) examined graduation rates of engineering students with respect to Calculus I grades at the university level. They found that 79.3% of students who earned an A graduated with a degree in engineering, 71.9% of students who earned a B graduated, 52.1% of students who earned a C graduate, and only 23.5% of students who earned a grade lower than a C graduated. It is important to note, however, that the relationship between grades and retention documented by Van Dyken et al. (2015) and Gardner et al. (2007) was based on a one-year retention rate. Students who do well in their first course, irrespective of the course, would likely have no reason to doubt their potential for graduation. Similarly, by considering only grades for students in Calculus I (Yoon et al., 2014), it is not clear how these rates compare across different courses. Studies considering the relationship between other mathematics courses and eventual graduation would enhance our understanding of the patterns associated with courses and grades.

Krause et al. (2015) examined the level of mathematics courses that engineering students took in their first-year in relation to the course grade to predict persistence to graduation as they tracked a cohort of students for six years. They grouped students into three levels based on their first mathematics course: (a) those who took Calculus I; (b) those who took a course above Calculus I; (c) and those who took a course below Calculus I. They found that regardless of the first-year course, 42% of students who earned an A or B leave the program within six years, 75% of students who earn a C leave, and 84% of students who earn a D, F, or W leave. After controlling for grade, course level was also a significant predictor of persistence to completion of an engineering degree. Approximately 71% of students who took a class below Calculus I switched programs, 50% of students who took Calculus I switched, and only approximately 28% of students who took a course above Calculus I switched programs within six years. From this study, we see that both the mathematics course and the level of success in the course relate to degree completion.

In sum, there are many factors that relate to student retention and degree completion in engineering programs: student gender, interest, identification, and self-efficacy (Besterfield-Sacre et al., 1997; Geisinger & Raman, 2013; Jones et al., 2016; Levin & Wyckoff, 1995); standardized test scores such as SAT and ACT math scores and overall GPA (Honken & Ralston, 2013; Levin & Wyckoff, 1988, 1995; Pembridge & Verleger, 2013; Yoon et al., 2014); and mathematics readiness and preparation (Gardner et al., 2007; Geisinger & Raman, 2013; Redmond-Sanogo et al., 2016). However, within the research literature, the first mathematics course a student takes as part of their engineering program and the grade they receive in this course seem to have a prominent predictive relationship with retention and degree completion. Most studies to date investigating mathematics courses and grades have only considered retention rates preceding graduation (e.g., one-year retention) or examined the course without including the course grade. Bowen et al. (2019) did consider degree completion as their outcome measure when investigating its relationship with first-year mathematics course but considered overall GPA after one year as a predictor instead of the specific grade in the mathematics course. Krause et al. (2015) considered both grade and course as predictors of degree completion; however, their study focused primarily on this success relative to Calculus I without specifically examining the relative success rates for other mathematics courses. Budny et al. (1998) did consider three first-year mathematics courses and the grades in these courses and documented an interesting pattern associated with grades, courses, and retention: students who earned an A in a Precalculus course had a similar retention rate as students earning a B in Calculus or a C in a more advanced mathematics course. By further investigating the relationship among the first-year mathematics course, grade earned in that course, and the relative likelihood of graduating with an engineering degree, the study reported here may confirm similar patterns and uncover additional patterns of success that can be beneficial for supporting students who are not ready to begin their first-year mathematics study with Calculus.

3 | PURPOSE OF STUDY

Prior research has documented a relationship between students' persistence in pursuing an engineering degree and their early experiences in undergraduate mathematics courses. Researchers have documented relationships between the first undergraduate mathematics course taken and retention as well as grade in the first mathematics course and retention. However, what is missing from the literature are extensive studies focusing on the interplay among students' first mathematics course in college, the grade they receive in that course, and their retention in an engineering major to graduation. To address this gap in the literature, the purpose of this study was to investigate the relationship among the first undergraduate mathematics course a student completes, the grade they earn in this course, and the likelihood of graduating with a degree in engineering within six years. In particular, this study investigates the interrelationship between course and grade by estimating and examining the overall relative likelihood of a student graduating with a degree in engineering based on different course and grade combinations. The following research questions guide our research design and analyses: Given a student's first mathematics course in college and final grade earned in the course, what is the likelihood that they graduate with a degree in engineering within six years? How does the likelihood compare based on the type of course and grade in the course?

4 | METHOD

4.1 | Participants

We obtained data for this study from the Office of Registration and Records at a medium-sized Midwestern public university. The data included first-year students admitted to the College of Engineering from Fall 2005 to Fall 2009. The students in this study represented a traditional fall first-year admittance into an engineering program. Therefore, the data did not include students who had transferred from another university. Overall, we began the study with a sample of 1579 students from five consecutive cohorts of first-year students enrolled in one of the engineering degree programs. This sample was reduced to 1504 because 73 students did not take a mathematics course in their first-year, and two students audited their first mathematics course and did not receive a grade.[1]

We were unable to obtain specific demographic data for the students in this study. However, we were able to obtain aggregate enrollment demographic data from the Office of Registration and Records for the years 2005–2009 for first-time, first-year students in engineering. The percentage of the population who self-identified as female was 16.1%, 15.0%, 15.5%, 16.6%, and 17.3%, respectively. The percentage of the student population who self-identified as White was 96.5%, 95.8%, 94.5%, 93.6%, and 93.0%, respectively. The percentage of the student population who self-identified as Asian was 1.4%, 1.3%, 2.3%, 3.7%, and 2.5%, respectively. The self-identification of the remainder of the student population was too small to report specifically, but the additional options to choose from were American Indian, Black, Hawaiian, Hispanic, or Not Specified. In 2021, approximately 40% of the student population enrolled comes from in-state. Based on a report from the *New York Times* ("Economic Diversity," 2017), the median family income for students at this university is $100,000.

4.2 | Engineering degree programs

The university from which the data were collected offers eight engineering degree programs that lead to eligibility for the Professional Engineers License: Agricultural and Biosystems Engineering, Civil and Environmental Engineering (Civil), Computer Engineering, Construction Engineering, Electrical Engineering, Industrial Engineering and Management (Industrial), Manufacturing Engineering, and Mechanical Engineering. The first-year plan of study for the eight programs includes similar course requirements and recommendations, including Calculus I, Calculus II, Chemistry, English Composition, and Communication/Public Speaking, an Introduction course to the particular program, and sometimes a general education course or a course specific to the major.

4.3 | Measures

4.3.1 | Degree completion

We created a degree completion variable based on whether a student graduated with a degree in engineering from one of the eight engineering programs. We only considered students who were admitted to one of the eight degree programs as a first-semester student. However, students who finished an engineering degree different from the one in which they were originally admitted were still considered to be an engineering degree completer. We were primarily interested in whether a student graduated with a degree in engineering and not how fast they completed the degree. Therefore, to control for time, we used a six-year graduation rate. For example, just based on scheduling constraints, students beginning in a mathematics course prerequisite or preparatory to Calculus I would require longer time to complete a degree. We believe that a six-year cap represented a reasonable expectation for completion time (ASEE, 2017).

For each student, the number of years to complete a degree was determined by the semester in which they were awarded their degree, including the summer term. If a student finished their degree in the summer, they were coded as having completed their degree within that academic year; if a student finished their degree at the end of the fall term they were coded with an additional half year. Therefore, if a student finished their degree in the summer of their sixth year, they were coded as completing a degree within six years.

4.3.2 | First mathematics course

For each of the eight engineering programs, *Calculus I* was listed as the first mathematics course in the first-year of the plan of study. However, we documented 16 different courses taken by students as their first mathematics course in their first-year. These 16 different course titles are listed in Table 1. A student's first mathematics course was determined by one or a combination of several of the following: ACT math sub-test score, SAT composite score, Pearson's

TABLE 1 Mathematics course variables and mathematics course titles

Course variable	Course titles
Precollege algebra	Elementary algebra Intermediate algebra
College algebra	College algebra
Finite mathematics	Finite mathematics
Trigonometry	Trigonometry
Precalculus	Precalculus
Linear algebra	Introduction to linear algebra[a,b] Basic linear algebra[a,b]
Applied calculus	Applied Calculus I Applied Calculus II
Calculus I	Calculus I[b]
Calculus II	Calculus II[b]
Advanced mathematics	Multivariate calculus[c,b] Calculus III[c,b] Introduction to differential equations[b] Introduction to abstract mathematics

[a]These courses have similar content; only one of the two courses can be taken for credit.
[b]Mathematics course required for engineering degree.
[c]These courses have similar content; only one of the two courses can be taken for credit.

My Math Algebra and Trigonometry Tests scores, COMPASS® Mathematics Test score, ACCUPLACER® elementary Algebra score, or a university-based math placement test score. Based on the course catalog, students who were not fully prepared to take *Calculus I* were advised to take *College Algebra* or *Trigonometry* or take *Trigonometry* along with *Calculus I* depending on their standardized mathematics test scores or the results of their mathematics placement exams given by the university. Other mathematics courses taken by students that might be considered prerequisite or preparatory before taking *Calculus I* included *Precalculus*, *Finite Mathematics*, *Applied Calculus I* and *II*, *College Algebra*, *Elementary Algebra*, and *Intermediate Algebra*.

Some students took a Linear Algebra course as their first mathematics course. Although this course would not be considered more advanced than Calculus, it was a required course listed on the engineering plans of study in most cases as a second-year course to be taken after Calculus I and II. Other required and more advanced mathematics courses taken by students as their first mathematics course included *Calculus II*, *Calculus III*, *Multivariate Calculus*, and *Introduction to Differential Equations*. Although not required for engineering, one student took *Introduction to Abstract Algebra* as their first course.

For students who took a mathematics course in their first semester of their first-year, we identified this course as their first mathematics course (unless the course was an individual study course). If a student did not take a mathematics course in their first semester, then we identified the mathematics course taken in their second semester as their first mathematics course. If a student did not take a mathematics course in their first year, then the student was not included in the analysis. If a student was enrolled in both a *Trigonometry* and a *Calculus I* course, then that student's first mathematics course was coded as Calculus I.

We examined the list of courses for similarity in content and number of students enrolled. We used these factors to create 10 course variables indicating a student's first mathematics course experience in their first-year (Table 1). Based on the course catalog, *Calculus III* and *Multivariate Calculus* have common content and only one of these courses can be taken for credit, and thus, these two courses were combined, along with *Differential Equations* and *Abstract Algebra* (these two courses together included only five students), into one course variable called advanced mathematics. The two Applied Calculus courses (*Applied Calculus I* and *II*) were combined to create one Applied Calculus course variable. The two Linear Algebra courses (*Introduction to Linear Algebra* and *Basic Linear Algebra*) were combined to create one Linear Algebra course variable. Finally, *Elementary Algebra* and *Intermediate Algebra* were combined to create a Precollege Algebra course variable. Although only a small number of students took *Finite Mathematics* as their first course, initially combining this course with another course did not seem appropriate based on course content. The remaining course variables retained their course title name (Table 1). Once we determined this list of course variables, we created 10 dummy-coded variables indicating a student's first mathematics course. The frequency and percentage for each course variable are presented in Table 2.

TABLE 2 Frequency and overall percentage of students by course and grade

	Grade						
Course	**A**	**B**	**C**	**D**	**F**	**Total**	**Course %**
Precollege algebra	3	9	7	5	7	31	2.06
College algebra	23	32	22	21	30	128	8.51
Precalculus	26	37	48	34	39	184	12.23
Trigonometry	55	82	65	59	67	328	21.81
Applied calculus	6	6	2	3	3	20	1.33
Calculus I	107	175	196	22	55	555	36.90
Calculus II	53	53	37	10	15	168	11.17
Linear algebra	5	4	9	6	3	27	1.80
Advanced math	21	22	8	3	2	56	3.72
Finite mathematics	6	0	1	0	0	7	0.47
Total	305	420	395	163	221	1504	
Grade %	20.28	27.93	26.26	10.84	14.69		

4.3.3 | Grade

We created a grade variable based on the student's final grade in their first mathematics course.We created five dummy-coded grade variables (A, B, C, D, F) indicating the student's grade. The frequency and percentage for each grade variable are presented in Table 2.

4.4 | Data analysis

To answer our research questions, we used logistic regression to estimate a model for students' rate of graduating with a degree in engineering given their first mathematics course and grade. The logistic regression analysis was carried out using the logistic procedure in SAS Version 9.4 (SAS Institute, 2012). We used the following model to examine these relationships:

$$\text{Logit}\,(Y) = \ln\left(\frac{\pi}{1-\pi}\right) = b_0 + \sum\nolimits_{g=1}^{4} b_g Grade_g + \sum\nolimits_{c=1}^{9} b_{c+4} Course_c, \tag{1}$$

where b_0 is the intercept; b_1–b_4 are the regression coefficients representing the log of the odds for the effects of earning a grade of A, B, D, or F, respectively, controlling for course; and b_5–b_{13} are the regression coefficients representing the log of the odds for the effects of taking a first course in (1) Precollege Algebra, (2) College Algebra, (3) Precalculus, (4) Trigonometry, (5) Applied Calculus, (6) Calculus II, (7) Linear Algebra, (8) Advanced Mathematics, or (9) Finite Mathematics, respectively, while controlling for grade. The dummy grade variable for a C and the dummy course variable for Calculus I are not included in the model, which make students earning a C in Calculus I the comparison group. This means that b_0 represents the log of the odds of graduating with a degree in engineering for students earning a C in Calculus I.

The statistical significance of the regression coefficients (log of the odds) associated with grade and course is determined using the Wald chi-square statistic and represents the effect associated with a particular grade or course. We also report the associated odds ratio (OR) for each coefficient (calculated as e^b) along with its 95% confidence interval (an interval not including 1 would indicate statistical significance of the odds ratios, consistent with the Wald statistic).

In addition to the log of the odds and odds ratio, we calculated the *conditional probabilities* associated with each combination of grade and course. Using these probabilities, we calculated relative risk, or relative probability, which compares the probability of success in graduating with a degree in engineering associated with earning a particular grade in a given course to success given a particular grade in another course. The use of conditional probabilities and relative risk helps to facilitate interpretations associated with the relative likelihood of success among courses and grades. A relative risk close to 1.00 would suggest no difference in the likelihood of one event over the other. In terms of effect size, values of 1.22, 1.86, and 3.00 are recommended as benchmarks for small, medium, and large effects of relative risk, respectively (Olivier et al., 2017).

To evaluate the logistic models, we considered indicators of overall model fit, indicators of goodness-of-fit, and measures of association between observed values and predicted probabilities (Peng et al., 2002). To evaluate overall model fit, we used the likelihood ratio, Wald test, and Score test that compare the model to a baseline model in which no predictors are included. Statistical significance of these tests indicates that the model including the grade and course variables is more effective than the baseline model. To evaluate goodness-of-fit, we used the Hosmer–Lemeshow (H-L) test (Hosmer & Lemeshow, 2000). This test examines how well the predicted probabilities from the model fit the observed outcomes across multiple levels of probability. A statistically significant H-L test indicates that overall, the predicted probabilities are significantly different from the observed outcomes, and thus, an insignificant test suggests good model fit to the data.

Further evaluation of the logistic model resulted from comparing the predicted probabilities based on the model with the actual observed outcomes. In other words, to validate the predicted probabilities, we examined whether high-predicted probabilities align with observed events (i.e., graduating with a degree) and low probabilities align with nonevents. We report two measures of association to validate this level of agreement. The *c*-statistic, or concordance statistic, is a measure of association that represents the probability that given all pairs of students, one who graduates with a degree in engineering and the other who does not, the student who graduated with a degree in engineering has a higher predicted probability than the student who did not graduate with a degree in engineering. The *c*-statistic ranges from 0.5 to 1, with a probability closer to 0.5 suggesting that the predictions are no better than chance and a value of 1 representing perfect prediction. Hosmer and Lemeshow (2000) suggest that values between 0.7 and 0.8 represent relatively good model fit and that values greater than 0.8 represent strong model fit. Similar to *c*, the gamma statistic, or *G*, is another measure of association that compares concordant and disconcordant pairs but does not include pairs with tied rankings. *G* represents the percent difference in concordant and disconcordant pairs and can be interpreted in terms of the percent reduction in error in classification based on the logistic model compared with chance alone.

5 | LIMITATIONS

The data for this study were collected from a single institution. Therefore, we caution broad generalizations beyond this single institution. Furthermore, the demographics of this institution may not be representative of other medium-sized, public research universities with an engineering school. However, we highlight the large number of students in this institution pursuing an engineering degree who begin their study of mathematics with Trigonometry and Precalculus and suggest that our findings might provide useful insights for other universities as they support students beginning their study of mathematics in courses such as these two.

We also recognize that our focus on course and grade neglected other important documented factors associated with graduation rates in engineering. However, our goal was to extend prior research that focused specifically on students' first mathematics course and the graduation patterns associated with success in this course. The patterns documented in this study provide important evidence for future research to examine the transferability of results with other replication studies that include other potential predictors of graduation success.

While grades are the standard used in higher education to signify achievement, grades are potentially affected by many factors other than what students learn in a particular course, such as classroom climate, instructional methods, instructor support, or peer support. A low grade could be based on factors outside the control of the student, including systematic bias against women and racially minoritized populations (Stewart, 2013). Thus, we acknowledge the potential bias associated with grades as a measure of success in students' first mathematics course.

Individual demographic data were not available to us; therefore, we were unable to link individual students' demographic information to their mathematics course, grade, and graduation data to investigate these relationships. In addition to demographic variables, we encourage researchers to consider other factors (e.g., classroom and academic climate; student motivation, identification, confidence, and interest; and advising and academic support) in future studies to further enhance our understanding of the findings from this study.

6 | RESULTS

6.1 | Descriptive statistics

We present the descriptive statistics for first mathematics course and grade, graduation rates, and average number of years to graduate by course in Tables 2–4. The graduation rate across the different programs ranged from a high of 66.67% in Manufacturing Engineering to a low of 39.58% for Computer Engineering (Table 3). Overall, 55.25% of the students in the sample graduated with an engineering degree within six years (Table 3). This rate is similar to the national average (ASEE, 2017).

Considering students' first mathematics course (Table 2), the largest percentage of students (36.90%) completed Calculus I as their first mathematics course. Trigonometry was the second most completed first mathematics course (21.81%), followed by Precalculus (12.23%) and Calculus II (11.17%). Considering the mathematics courses found on the plans of study for the eight engineering degree programs, 53.59% of the students completed one of these mathematics courses (or a higher level; recall that one student completed a more advanced mathematics course that was not required) as their first mathematics course. However, this means that 46.41% of the students admitted into an engineering program began their study of mathematics in a prerequisite or preparatory course to Calculus I. Considering grade, the largest percentage of students in the sample earned a grade of B in their first mathematics course (27.93%), followed by a C (26.26%), and an A (20.28%). Only 10.84% of the students earned a D, and 14.69% earned an F.

TABLE 3 Graduation rate for engineering degrees within six years

First-year program	N	Graduate within 6 years: Frequency	%
Mechanical	378	231 (20)	61.11
Civil	328	192 (3)	58.54
Industrial and management	295	127 (53)	43.05
Electrical	254	152 (8)	59.84
Computer	96	38 (6)	39.58
Agricultural and biosystems	81	46 (2)	56.79
Construction	51	31 (1)	60.78
Manufacturing	21	14 (0)	66.67
Total	1504	831 (93)	55.25

Note: Numbers in parentheses represent the number of students who graduated with an engineering degree in a program other than their first-year program.

TABLE 4 Average number of years to graduate in engineering by first mathematics course

Course	*M*	SD	MIN	MAX
Precollege algebra	5.83	0.26	5.5	6.0
College algebra	5.10	0.42	4.5	6.0
Precalculus	4.96	0.58	4.0	6.0
Trigonometry	4.97	0.61	3.5	6.0
Applied calculus	5.00	0.45	4.5	6.0
Calculus I	4.53	0.53	3.5	6.0
Calculus II	4.37	0.50	3.0	6.0
Linear algebra	4.04	0.70	2.5	5.5
Advanced math	4.11	0.55	3.0	6.0
Finite mathematics	4.83	0.52	4.0	5.5
All courses	4.63	0.62	2.5	6.0

Note: Time to degree completion was limited to 6 years.
Abbreviations: *M*, mean; MIN/MAX, minimum/maximum number of years to graduate; SD, standard deviation.

The average number of years to graduate with a degree in engineering by first mathematics course is presented in Table 4. On average, it takes approximately 4.5 years to graduate ($M = 4.63$) with a degree in engineering from this university. On average, students beginning in Calculus I also take about 4.5 years to graduate ($M = 4.53$). Considering other courses, overall, it would appear that the average number of years to graduate tends to coincide with the number of semesters removed from starting with Calculus I. For example, students beginning with Trigonometry ($M = 4.97$) or Precalculus ($M = 4.96$) take, on average, approximately 5 years to graduate with a degree in engineering, one semester longer than students beginning with Calculus I.

6.2 | Logistic model predicting student graduation rate

To address our research question regarding the relationship between students' graduation rate in engineering and students' first mathematics course and the grade earned in the course, we estimated a logistic model. The model regressed graduation rate on grade and course variables. We used the Calculus I variable as the course comparison variable in the model; that is, all course effects were in comparison to students whose first course was Calculus I. We used the grade C variable as the grade comparison variable. The results from the model are presented in Table 5.

TABLE 5 Logistic regression analysis of graduating with a degree in engineering relative to first mathematics course taken and the grade in that course

Predictor	B	SE	Wald's χ^2	df	p	e^B (OR)	95% WCI
Constant	0.49	0.12	16.27	1	<.0001	NA	NA
Grade							
Grade A	1.10	0.18	36.05	1	<.0001	3.00	(2.10, 4.30)
Grade B	0.57	0.15	13.68	1	.0002	1.77	(1.31, 2.38)
Grade D	−0.43	0.21	4.31	1	.0380	0.65	(0.44, 0.98)
Grade F	−1.99	0.23	73.37	1	<.0001	0.14	(0.09, 0.22)
Course							
Precollege algebra	−1.96	0.49	16.27	1	<.0001	0.14	(0.05, 0.37)
College algebra	−1.98	0.25	60.91	1	<.0001	0.14	(0.08, 0.23)
Precalculus	−0.79	0.19	16.95	1	<.0001	0.45	(0.31, 0.66)
Trigonometry	−0.53	0.16	11.03	1	.0009	0.59	(0.43, 0.81)
Applied calculus	−0.46	0.51	0.82	1	.3645	0.63	(0.23, 1.71)
Calculus II	0.33	0.22	2.39	1	.1218	1.40	(0.92, 2.13)
Linear algebra	0.73	0.49	2.21	1	.1371	2.07	(0.79, 5.38)
Advanced math	0.75	0.40	3.58	1	.0585	2.12	(0.97, 4.61)
Finite math	0.41	1.10	0.14	1	.7086	1.51	(0.17, 13.17)

Test	%	χ^2	df	p
Overall model evaluation				
Likelihood ratio test		414.24	13	<.0001
Score test		370.07	13	<.0001
Wald test		280.30	13	<.0001
Goodness-of-fit test				
Hosmer–Lemeshow		6.92	8	.5457
Gamma	58.90			
c-Statistic	78.20			

Abbreviations: *B*, regression coefficient; df, degrees of freedom; OR, odds ratio; *p*, *p*-value; SE, standard error of *B*; WCI, Wald confidence interval.

The final model (Table 5) was found to significantly improve model fit when compared with a null model (likelihood ratio: $\chi^2(13) = 414.24$, $p < .0001$; additional results from the Wald and Score test can be found in Table 5). Further evidence of a good model fit to the data was based on a nonsignificant H-L test ($\chi^2(8) = 6.92$, $p = .55$), indicating no statistical difference between observed values and predicted probabilities from the model. In addition, two measures of association between predicted probability and observed values, $G = 58.90\%$ and $c = 78.20\%$, also indicated good model fit.

Based on the model (Table 5), when controlling for students' first mathematics course, on average, students' grade in their first mathematics course is a positive and statistically significant predictor of subsequent graduation with a degree in engineering. That is, higher grades are related to an increased likelihood of graduating with an engineering degree, and lower grades are related to a decreased likelihood of graduating with an engineering degree. For example, in their first mathematics course, students who earned a grade of A ($B = 1.10$, standard error [SE] $= 0.18$, Wald statistic $= 36.05$, $p < .0001$; OR $= 3.00$) or B ($B = 0.57$, SE $= 0.15$, Wald statistic $= 13.68$, $p = .0002$; OR $= 1.77$) were more likely to graduate with a degree in engineering than students who earned a C. Similarly, students who earned a grade of D or an F in their first mathematics course were less likely to graduate with a degree in engineering than students who earned a C (Table 5). Furthermore, the difference in the log of the odds for earning an A (1.10) compared with earning a B (0.57) also represents a statistically significant increase in the likelihood of graduating with a degree in engineering ($B = 0.53$, SE $= 0.18$, Wald statistic $= 8.36$, $p = .004$; OR $= 1.70$). Similarly, earning an *F* significantly decreased the likelihood of graduating with a degree in engineering when compared with earning a D ($B = -1.57$, SE = 0.27, Wald statistic = 34.10, $p < .0001$; OR = 0.21).

When controlling for grade, the level of the course is a statistically significant predictor of graduating with a degree in engineering. Specifically, on average, students who took Calculus I as their first mathematics course were more likely to graduate with a degree in engineering than students who took Precalculus, Trigonometry, College Algebra, or a Precollege Algebra course as their first mathematics course (Table 5). Students who took Calculus II as their first mathematics course were not statistically more likely to graduate with a degree in engineering than students who took Calculus I. Students who took a more advanced mathematics course as their first course (i.e., Calculus III, Multivariable Calculus, Differential Equations, or Abstract Algebra) were only marginally more likely to graduate with a degree in engineering than students who took Calculus I as their first course. The likelihood of graduating with an engineering degree for students whose first course was Linear Algebra was also not statistically different from those taking Calculus I. Finally, some students chose to take three other courses that are not prerequisites or requirements for an engineering degree—Finite Mathematics, Applied Calculus I, or Applied Calculus II (recall that the two applied Calculus courses were combined into one categorical variable titled Applied Calculus)—as their first mathematics course. Taking Finite Mathematics or Applied Calculus was also not found to change the likelihood of graduating with a degree in engineering compared with students who took Calculus I.

Comparing the log of the odds of students taking Trigonometry ($B = -0.53$) with those taking Precalculus ($B = -0.79$) as their first course found no statistical difference in the likelihood of graduating with a degree in engineering ($B = 0.26$, SE = 0.21, Wald statistic = 1.65, $p = .20$; OR = 1.30). However, on average, students taking College Algebra were less likely to graduate with a degree in engineering than students taking either Trigonometry ($B = -1.45$, SE = 0.26, Wald statistic = 30.33, $p < .0001$; OR = 0.23) or Precalculus ($B = -1.19$, SE = 0.29, Wald statistic = 17.33, $p < .0001$; OR = 0.30).

6.3 | The likelihood of graduating based on the interrelationship between course and grade

To further understand the interrelationship between course and grade and the likelihood of graduation, we used our logistic model to calculate the conditional probabilities associated with specific course and grade combinations. These probabilities are presented in Table 6 and can be used to provide estimates of likelihood for specific grades and courses as well as for calculating measures of relative risk, or relative probability, associated with different scenarios involving courses and grades. For example, using the specific coefficient estimates associated with a grade of A (1.10) and the value for the intercept associated with being in Calculus I (0.49) (Table 5), the probability of a student graduating with a degree in engineering given that they earned an A in Calculus I would be equal to $\frac{e^{(0.49+1.10)}}{1+e^{(0.49+1.10)}} = 0.83$. Therefore, based

TABLE 6 Conditional probabilities of graduating with a degree in engineering by first course and grade combination

	Grade				
First course	A	B	C	D	F
Advanced math	0.91	0.86	0.78	0.69	0.32
Linear algebra	0.91	0.86	0.77	0.69	0.32
Finite mathematics	0.88	0.81	0.71	0.62	0.25
Calculus II	0.87	0.80	0.70	0.60	0.24
Calculus I	0.83	0.74	0.62	0.52	0.18
Applied calculus	0.76	0.65	0.51	0.40	0.12
Trigonometry	0.74	0.63	0.49	0.39	0.12
Precalculus	0.69	0.57	0.43	0.33	0.09
College algebra	0.40	0.28	0.18	0.13	0.03
Precollege algebra	0.41	0.29	0.19	0.13	0.03

Note: Shading represents magnitude of probability: No shading is greater than or equal to 60%; medium shading is 50%–59%; dark shading is less than 50%.

on the model, on average students who earn an A in Calculus I as their first mathematics course have an 83% chance of graduating with a degree in engineering. However, using this model, we also find that a student who takes Calculus I and earns a D would have only a 52% chance of graduating with a degree in engineering, a relative probability of 1.60. That is, students who receive an A in Calculus I are 1.60 times more likely to graduate with a degree in engineering than students who earn a D. This represents a substantial difference in likelihood. By examining the probabilities in the table, we can better understand the patterns associated with the relative importance of grades and courses in students' graduation success.

In an absolute sense, consistent with the overall findings of the model, we see that students who take Calculus I or a higher-level course as their first mathematics course and earn a passing grade are, in general, more likely to graduate with a degree in engineering (i.e., greater than a 50% chance). On the other hand, not surprisingly, students who do not earn a passing grade in their first mathematics course (no matter the course) are, overall, unlikely to graduate with a degree in engineering even for students starting in more advanced mathematics courses.

However, from a relative point of view, which is not as apparent from the general model, students who earn a grade of A or B in Trigonometry are no less likely to graduate with a degree in engineering than students who earn a B or C in Calculus I. Moreover, students who earn an A in Precalculus are more likely to graduate with a degree in engineering than students who earn a C in Calculus I. When comparing students who earn an A in Calculus I with students who earn an A in Precalculus, the relative risk is only 1.20, an effect size that does not reflect a substantially meaningful difference (Olivier et al., 2017). What seems to be the case is that the relative risk associated with course becomes more apparent as students are less successful in the course. That is, when students perform more poorly in lower-level courses, they tend to become relatively more likely not to graduate with a degree in engineering. Nevertheless, for students who do well in Precalculus or Trigonometry, the likelihood of graduating with a degree in engineering remains relatively similar to students who begin with Calculus I.

Based on the model, from an absolute sense, students who begin their study of mathematics with a College Algebra course or a Precollege Algebra course have less than a 50% chance of graduating with a degree in engineering even if they receive an A in the course, 40% and 41%, respectively. Relatively speaking, students who earn an A in Calculus I are 2.08 times more likely to graduate with a degree in engineering than students who earn an A in College Algebra, and even students who earn an A in Precalculus are 1.73 times more likely to graduate. From a positive perspective, the model predicts that approximately 40% of the students who earn an A in their first Algebra course graduate with a degree in engineering. However, the model also suggests that if the grade earned is lower than an A, then the likelihood of success greatly diminishes, more so than with upper-level courses. For example, recall that the relative risk for earning an A versus a D in Calculus I was 1.60; when considering College Algebra, this relative risk for an A versus a D is increased to 3.08. The influence of nonsuccess in lower-level courses is also evidenced in the fact that students who take an advanced mathematics course as their first mathematics course and earn an F are still more likely to graduate with a degree in engineering than students whose first course is one of the Algebra courses and they earn a B.

There were a small number of students ($n = 27$) who took a Linear Algebra course as their first mathematics course. Overall, although there was no statistical difference in the likelihood of graduating with a degree in engineering for these students compared with those taking Calculus I, the absolute probabilities are all higher at each grade level and also higher than students taking Calculus II. The Linear Algebra courses carry a course number that would seem to indicate that it is a lower-level mathematics course when compared with Calculus. Linear Algebra is a required course for all engineering majors and is most often a new area for students entering their first-year of college. Students usually take this course in their second year after completing Calculus I and Calculus II. Therefore, taking Linear Algebra as a first course is likely an indication that the students have entered the program with credit for the first two Calculus courses. It follows that the likelihood of these students graduating with a degree in engineering would be relatively similar to students who have taken Calculus I and/or Calculus II.

There were also a small number of students ($n = 20$) who took one of the Applied Calculus courses as their first mathematics course. Interestingly, these students, overall, were not less likely to graduate with a degree in engineering than students who took Calculus I as their first course. Further investigation would be necessary to identify the course trajectory of students starting with an Applied Calculus course, but it may be that these courses served as a test case for students who were hesitant about taking Calculus but upon success, were willing to proceed to Calculus I or even jump to Calculus II.

6.4 | Graduation rates by engineering program

In addition to the estimation of the overall model, we further examined the interrelationship between course and grade by individual engineering programs. These follow-up analyses served as a robustness check of the findings from the overall model. In other words, we wanted to determine if the overall findings were consistent across programs or if the overall findings were somehow influenced by aggregation bias or other confounding variables. Finding similar patterns across different programs would give us further confidence that the overall findings are robust and meaningful. This analysis also puts a focus on the comparison among Calculus I, Trigonometry, and Precalculus (further discussion of the program analyses appears in the Appendix).

Four engineering programs had a sufficient sample of students to estimate valid models: Civil, Electrical, Industrial, and Mechanical. Comparing the four programs, overall, patterns in graduation rates associated with different course and grade combinations were similar to those found in the overall model (details can be found in the Appendix, Table A1). Across the four programs, students who completed Calculus I or a more advanced mathematics course with a passing grade were, on average, more likely to graduate with a degree in engineering (with two exceptions, earning a D in Calculus I for Civil [35%] and Electrical Engineering [49%]). Similar to the overall trend, this was also true for students who earned a grade of A or B in Trigonometry or Precalculus. Across the programs, in general from a relative point of view, students who do very well in Trigonometry or Precalculus (i.e., earn a grade of A or B) were as likely to graduate with a degree in engineering as many students who earn a B or C in Calculus I. Moreover, in all four programs, students earning an A in Trigonometry or Precalculus were more likely to graduate with a degree in engineering than students who earned a C in Calculus I.

Overall, the patterns associated with different course and grade combinations found across different programs were similar to the overall trends. In particular, students who are successful in Trigonometry and Precalculus are relatively as likely to graduate with a degree in engineering as students who begin in Calculus I. These findings provide evidence of the robustness of the findings from the overall model with little evidence of aggregation bias. However, it is important to point out that there were differences across programs in absolute graduation rates by courses, with the effects being strongest in Mechanical Engineering and Industrial Engineering but practically nonexistent in Civil and Electrical Engineering.

7 | DISCUSSION

Previous research has indicated the importance of students being able to take Calculus I in the first-year of their engineering program in order to successfully persist to graduation with an engineering degree (e.g., Bowen et al., 2019; Krause et al., 2015; Levin & Wyckoff, 1995). Other research has indicated that the level of success in mathematics in the first-year is an important predictor of retention in engineering (e.g., Budny et al., 1998; Krause et al., 2015; Levin & Wyckoff, 1995). The goal of this study was to examine the first mathematics course experience in more detail to determine just how important it is to start with Calculus I compared with other courses and to determine the relationship between success in these different courses and students' eventual graduation with a degree in engineering. To control for time to graduation, we considered a six-year window for graduation (ASEE, 2017). Thus, our findings focus on graduation as the goal and not on the amount of time to graduate.

Overall, our study confirmed what had been previously documented in other studies about the significance of taking Calculus I as a first mathematics course. That is, controlling for grade, students taking Calculus I or a more advanced mathematics course as their first mathematics course were more likely to graduate with a degree in engineering than students who did not begin with at least Calculus I. Furthermore, overall, controlling for course, students who were more successful in their first mathematics course based on their final grade in the course were more likely to graduate with a degree in engineering. However, we also considered the interrelationship between grade and course, which offered a clearer picture of the influence of students' first mathematics course experience on their eventual chances of graduating with a degree in engineering.

Although taking and passing Calculus I (or a more advanced course) is a strong predictor of graduating with a degree in engineering, overall success plays an important role in this predication as well. For example, students who earn an A in Calculus I were 1.60 times more likely to graduate with a degree in engineering than students who earned a D in Calculus I. Not surprisingly, this finding suggests that while it is important to be ready upon entering college to enroll in higher-level mathematics courses in the first-year, overall success in these courses is also important in predicting later success.

Similarly, based on the results of this study, it appears that the importance of taking Calculus I as the first mathematics course for successfully graduating with a degree in engineering may not be as clear-cut as has been previously portrayed. In this study, overall, students who were successful in Trigonometry, as indicated by earning an A or B, were as likely to graduate with a degree in engineering as students who earned a B or C in Calculus I, respectively. In fact, students who earned an A in Calculus I were only 1.12 times more likely to graduate with a degree in engineering than students who earned an A in Trigonometry; relatively speaking, this does not represent a substantially meaningful difference. Even students who earned an A in Precalculus were as likely to earn a degree in engineering as students who earned a C in Calculus I. Moreover, for those students earning an A, students in Calculus I were only 1.20 times more likely to graduate with a degree in engineering than students in Precalculus; again, relatively speaking, the eventual impact of taking Calculus I over Precalculus for students who are successful is almost nonexistent. Relatively speaking, students who do well in Trigonometry or Precalculus are as likely to graduate with a degree in engineering as many students who begin in Calculus I (cf. Budny et al., 1998). Perhaps, for those students who did well in Precalculus and Trigonometry and did go on to successfully graduate with a degree in engineering, it was never a question of talent or ability to succeed but an issue of opportunity and support.

These findings suggest that having to start with a Precalculus course may not be a "death sentence" for engineering students (cf. Van Dyken & Benson, 2019) but instead an avenue for potentially increasing participation in engineering. Enrollment in courses such as Precalculus or Trigonometry is often viewed from a deficit perspective signaling a need for remediation (Hsu & Bressoud, 2015); however, another view might advocate for these courses as a way to increase access and opportunity for many talented students who did not have the opportunity to prepare for Calculus in high school (Mathematical Association of American and the National Council of Teachers of Mathematics [MAA/NCTM], 2012). Across the United States, students' opportunities to study mathematics often depend on where they attend high school and the availability of courses within their high school or middle school (Mendes & Duncombe, 2020; Norwicki, 2016; Oakes et al., 2003). Moreover, access to advanced mathematics courses is systematically differentiated based on the socioeconomic, racial, and ethnic makeup of US high schools (Battey, 2013; Ma & Willms, 1999; Mendes & Duncombe, 2020; Norwicki, 2016; Oakes et al., 2003). Within high schools, curricular tracking in mathematics further differentiates access to advanced mathematics courses (Moses & Cobb, 2001; Oakes, 1985, 1990; Oakes et al., 1992, 2003). For this reason, many students entering college have not had equal opportunities to study advanced mathematics (Mendes & Duncombe, 2020; Norwicki, 2016; Oakes et al., 2003). The results from this study, coupled with what the literature has shown about differential access to certain math courses across high schools, point to supporting students' success in Precalculus courses as a potential means to broaden participation (with the caveat that taking additional mathematics courses could potentially lengthen time to graduation). However, based on this study, when compared with students starting in Calculus I, students starting in Precalculus or Trigonometry, on average, took only one more semester to complete their degrees (Table 4).

It is necessary to emphasize the importance of success in these courses in order to persist to graduation; it is not enough just to complete the courses (Table 6). The importance of this success has further implications for improving the quality of the overall course experiences in terms of curriculum and instruction offered to students in Precalculus and Trigonometry (Carlson et al., 2015, 2019; Carlson & Rasmussen, 2008; Hsu et al., 2008; MAA/NCTM, 2012; Rasmussen et al., 2019). Furthermore, with evidence that students can successfully complete a degree after starting in a non-Calculus course, it may be necessary to modify the program curriculum to better accommodate alternative curricular routes. The content might be embedded in alternative first-year experiences that support students who may not be fully prepared mathematically for a first course in Calculus. For example, one university developed an engineering course, *Introductory Mathematics for Engineering Applications*, which uses an application-oriented, hands-on approach to teaching the essential mathematics skills needed for subsequent engineering courses (Klingbeil & Bourne, 2015). Related to alternative routes, an interesting finding in this study, which would require further research to understand fully, is the relatively high likelihood of graduating with a degree in engineering for students taking courses such as Finite Mathematics or Applied Calculus I or II. The number of students in this study taking these courses was small, but a pattern of success was evident. Perhaps, these courses provide some students with a more interesting or accessible entry into the study of mathematics in their first–year or provide a needed precursor to taking Calculus I. Further study to understand viable alternate routes through the first-year mathematics curriculum could be beneficial for advising certain populations of students.

The study reported here confirms the "long row to hoe" for students who begin their mathematics experience in College Algebra or a Precollege Algebra course. From a positive perspective, on average, the probability is approximately 40% for students earning an A in an Algebra course eventually graduating with a degree in engineering. However, for students earning less than an A, there is a significant drop in the likelihood of graduating with a degree in engineering, much more so than for the higher-level courses. Again, additional supports may be necessary for students entering college unprepared to take a course beyond College Algebra to broaden participation of students who would otherwise not persist in an engineering major.

The findings from this study also have important implications for the overall tutoring, training, and advising of engineering students in their academic and self-regulation strategies (Budny et al., 1998; Ohland et al., 2004; Rasmussen et al., 2019; Van Dyken & Benson, 2019). One of the primary reasons why students do not persist through an engineering program is their readiness for the high level of rigor required by engineering courses, including success in mathematics (College Transitions, 2018). If academic advisors are more aware of the success factors associated with mathematics course selection and grade earned, both the advisor and student will be more informed in their decisions about the student's academic future. Advising students to take a course at a lower level than Calculus I first to strengthen their mathematics may benefit the student more in the long run than trying to start with Calculus I and not doing well (Budny et al., 1998; Ohland et al., 2004). Moreover, universities might focus on the development of alternative routes to strengthen the mathematical skills and applications to engineering for students who are not fully prepared upon entering college (e.g., Klingbeil & Bourne, 2015). Although helping students explore their major has been identified as one of the key duties of academic advising (McGlothlin Lester, 2019), this study shows that mathematics course selection plays a critical role in degree completion of that particular engineering major.

8 | CONCLUSION

Graduation rates in engineering continue to be a concern in higher education. Prior research has pointed to student experiences in first-year mathematics courses as a key predictor of retention and graduation in engineering. Much of this research has focused on the positive relationship between students' access to and success in Calculus and retention and graduation in engineering. Findings from this study confirm this relationship. However, these findings also show that students who are successful in Precalculus or Trigonometry courses as their first mathematics course in college are as likely to graduate as some students who begin in a Calculus course. This evidence helps to dispel the myth that beginning with higher-level mathematics courses is always the optimal course-taking strategy when pursuing an engineering degree. Moreover, given the differential access to higher-level mathematics courses in high school, this finding provides motivation to focus on supporting students' success in these courses as a way to broaden participation in engineering programs.

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ENDNOTE

[1] Of these 75 students, 42 (56%) did not graduate with a degree from this university; 14 (19%) graduated with a degree in engineering within six years; 3 (4%) graduated with a degree in engineering but took longer than six years; and 16 (21%) graduated with a degree in a program other than one of the eight engineering programs.

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APPENDIX

Analysis of graduation rates for course and grade combinations by engineering program

Here, we provide additional background information for the examination of the interrelationship between course and grade for individual engineering programs. Graduation rates by program are presented in Table 3. Students who graduated with a degree in engineering most often graduated from their first-year program, with the exception of Industrial Engineering for which 41% graduated in a different program, primarily Mechanical or Civil Engineering. Therefore,

TABLE A1 Conditional probabilities of graduating with a degree in engineering by course and grade combination by engineering program

	Grade				
Course	**A**	**B**	**C**	**D**	**F**
Civil engineering					
Other	0.96	0.86	0.81	0.61	0.54
Calculus II+	0.92	0.75	0.67	0.44	0.36
Calculus I	0.89	0.68	0.59	0.35	0.28
Trigonometry	0.85	0.60	0.50	0.27	0.22
Precalculus	0.83	0.57	0.47	0.25	0.20
Algebra	0.72	0.41	0.32	0.15	0.11
Electrical engineering					
Other	0.87	0.88	0.74	0.54	0.21
Calculus II+	0.78	0.79	0.60	0.38	0.13
Calculus I	0.85	0.85	0.70	0.49	0.18
Trigonometry	0.80	0.81	0.63	0.41	0.14
Precalculus	0.78	0.78	0.60	0.38	0.12
Algebra	0.21	0.21	0.10	0.04	0.01
Industrial engineering					
Other	0.87	0.78	0.66	0.65	0.24
Calculus II+	0.85	0.74	0.61	0.60	0.21
Calculus I	0.82	0.70	0.56	0.55	0.17
Trigonometry	0.68	0.52	0.37	0.36	0.09
Precalculus	0.68	0.52	0.37	0.36	0.09
Algebra	0.35	0.21	0.13	0.12	0.02
Mechanical engineering					
Other	0.89	0.80	0.62	0.57	0.09
Calculus II+	0.96	0.91	0.81	0.78	0.22
Calculus I	0.88	0.77	0.58	0.53	0.08
Trigonometry	0.69	0.51	0.30	0.26	0.03
Precalculus	0.67	0.49	0.28	0.24	0.02
Algebra	0.38	0.23	0.11	0.09	0.01

Note: Shading represents magnitude of probability: No shading is greater than or equal to 60%; Medium shading is 50%–59%; Dark shading is less than 50%.

using student first-year program for a program analysis seemed to be an acceptable unit of analysis although results from the Industrial Engineering program should be interpreted with the described caveat.

To accommodate the modeling for individual programs, several course variables were combined to create six dummy-coded course variables (Table 1 provides details for comparison): Algebra (Precollege and College), Trigonometry, Precalculus, Calculus I, Calculus II+ (Calculus II, advanced mathematics), and Other (Finite Mathematics, Linear Algebra, and Applied Calculus). In some cases, course variables were combined because of small numbers of students enrolled in particular courses within a particular program (e.g., Finite Math, Precollege Algebra, and Linear Algebra). In other cases, the combination of course variables was justified based on results from the overall model suggesting little difference across courses as compared with Calculus I (Table 5). Logistic models were estimated for all eight programs; however, the validity of the model fit based on the data from four of the programs was found to be questionable. This lack of model fit appeared to be due to the small number of students enrolled in the particular programs; these four programs were the four smallest programs based on first-year enrollment (Table 3). Thus, these four programs were not individually analyzed.

Using our logistic models, we again calculated the conditional probabilities associated with specific course and grade combinations by program (details are presented in Table A1). By examining the probabilities in Table A1 as well as relative risks associated with different pairs of courses, overall, we were able to evaluate whether the patterns across programs were similar to those for engineering students in general at the university (Table 6).